

AN APPROACH TO THE EFFICIENCY OF PRODUCTION DECISIONS IN MULTIPRODUCT FIRMS

**A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY**

**by
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to the

**DEPARTMENT OF HUMANITIES AND SOCIAL SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

AUGUST, 1986

Dedicated to

the memory of

my mother

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This is to certify that the thesis "An Approach to the Efficiency of Production Decisions in Multiproduct Firms" submitted by Mr. A.V. Raja in partial fulfilment of the degree of Doctor of Philosophy to the Indian Institute of Technology, Kanpur is a record of bonafide research work carried out under my supervision and guidance. The results embodied in the thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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Course Number	Course Name
H-Eco 740	Inter-Industry Economics
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M 541	Matrix Algebra and Linear Estimation
H-Eco 736	Industrial Organization and Policy
H-Eco 739	Price Theory
H-Eco 742	Seminars on an advanced topic in economic theory (the topic chosen was: Theory of the Firm in Economic Space)
M 502	Computer Programming
H-Eco 732	Econometrics

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- A.V. Raja

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SYNOPSIS

AN APPROACH TO THE EFFICIENCY OF PRODUCTION DECISIONS IN MULTIPRODUCT FIRMS

A thesis submitted in partial fulfilment of the requirements for a Degree of Doctor of Philosophy by A.V. Raja to the Department of Humanities and Social Sciences, Indian Institute of Technology, Kanpur, August 1986.

A prominent feature of the modern corporate business firm is the diversity of products that it offers on the market. Usually, such enterprises have technological and organizational structures that are both large and complex. The available literature on the theoretical characterization of product diversity by a single firm is scant. Moreover, economically efficient choices of products within a product line is not studied.¹ The focus is on the analysis of profit maximizing policy for a monopolist/monopolistic producer producing a product line) (Mussa and Rosen 1978; Itoh 1983; Katz 1984). The main insight from these studies is that the product line may be used by the firm to engage in imperfect price discrimination.¹ Spence (1976b, p. 226), while considering multiproduct firms briefly, felt that, "Products that are close substitutes will be produced by separate firms." Also, the effect of each product upon profits generated by other products of the firm tends to limit the number of products. None of these studies acknowledge that multiproduct production can emerge because of economies of scope.

¹Price discrimination has also been used as an explanation for multiproduct production (Clemens 1954).

Hence, the cost specification in these studies is incomplete. However, several useful insights offered by other studies on product variety in monopolistic markets appear to be amenable for extensions to the multiproduct firm. For instance, it was argued in Spence (1976a,b), that the firm would select products on the basis of the fraction of net potential surplus that can be converted into profit. As a result, firms would be biased against products with low elasticity of demand. Further, multiproduct firms are more likely to offer a range of complementary products.

The present study has three main objectives:

- a) To explicitly define the economically efficient levels of products within a product line,
- b) To characterize the cost advantages pertaining to multiproduct production, and
- c) To extend the basic insights obtained from the product diversity literature dealing with single product firms to the multiproduct firm.

The emergence of a multiproduct firm through internal expansion, rather than mergers, is the main focus of the present study. It is assumed that the availability of economies of scope is the main motivation for internal expansion. This necessitated the specification of significant cost-interrelationships between products in a product line. This was done using the concept of weak cost complementarity developed recently in the literature (Baumol, Panzar and Willig 1982). The contribution of the organizational structure to such cost advantages has been argued using the insights from Penrose (1959).

Sharkey (1982) and Williamson (1975). The possible demand interrelations that can occur between products has been taken into account explicitly. The benchmark of efficiency is defined as product quantities which enable full exploitation of cost advantages of joint production. The welfare maximising quantities of products unrelated in demand were found to correspond to cost efficiency choices only under specific assumptions on demand structures. These assumptions were retained throughout the rest of the analysis. The existence of demand interrelations precludes the firm from producing in a cost efficient manner. Specifically, substitution effects prevent the firm from exhausting economies of joint production. Complementarity effects, on the other hand, lead to production of such products even if cost-advantages are absent. The incorporation of the effect of outside competition on product choices enabled the extension of some of the results obtained in the product diversity literature to the multiproduct case albeit with additional qualifications. For example, product selection under a multiproduct setting would have economies of scope considerations in addition to the amount of surplus that can be converted into profits. Similarly, it is found that products will tend to be dropped from production lines whenever the infra-marginal revenues obtainable from the product are adversely affected. This is in addition to the 'twisting effect' on the demand curve described by Spence (1976b). In keeping with the literature, it is observed that there can be a bias against products with low elasticity of

demand even if it consists only of consumers who place a high valuation on the product.

The second aspect of the study is the analysis of vertical integration. The literature on transaction costs shows that firms would tend to vertically integrate their activities whenever it is cheaper to internalize rather than conduct the same activity by using the market. The many studies on vertical integration have not viewed integration as a process of internalizing a market transaction.³ Also, the number of studies on forward integration is more and backward vertical integration has received relatively less attention. The welfare impacts of vertical integration has been a source of much debate with little or no consensus on the issue. The Chicago school arguments perceive vertical integration as enhancing efficiency⁴ and leading to lower end product prices in some cases. On the other hand, vertical integration has been viewed as a source of heightened monopoly power.⁵ All these arguments are based upon a firm which is already vertically integrated. The process of integration itself plays a passive role.

³Dixit (1983) acknowledged the importance of transaction costs and their role in vertical integration decisions but did not incorporate this aspect in his study.

⁴Integration is argued to dissolve bilateral monopoly stalemates, eliminate double marginalization and minimize input substitution distortions.

⁵Strong bargaining positions in both the input and output markets and large capital requirements of integrated firms generally not within reach of potential entrants are cited as the main causes.

The present study develops a general framework to

- a) Explicitly define the welfare maximizing level of vertical integration and examine the effect on the end product prices, and
- b) To incorporate the transaction cost arguments to provide the motivation for vertical integration as argued by /

Williamson (1971). This is done through the cost function.

Vertical integration is viewed as an alternative organizational mode for the procurement of an input which is used for the production of a final product. The other feasible mode of input procurement is the purchase of the input on the market from a monopolistic input supplier. Initially, the presence of significant transaction costs of using the market was shown to be the main cause of vertical integration. The welfare maximizing level of integration for any given level of the final product was found to be that which minimized the total costs of production. The efficient level of integration was enhanced whenever there were additional economies of producing the input and output jointly. The end product prices were seen to reduce only under specific assumptions regarding the marginal costs of production of the final product. Also, the welfare maximizing level of integration did not necessitate the firm to be fully vertically integrated and hence such firms may not always enjoy monopoly power as is usually made out in the literature. When transaction costs are removed from the analysis then vertical integration becomes a more difficult proposition and a simple savings in the monopolistic price-cost margin seemed to be an unlikely reason to integrate. (Under specific assumptions about

economies of scale, full vertical integration was shown to be the economically efficient level of integration.

Both the above models of horizontal expansion and vertical integration have the following limitations. For the sake of analytical simplicity the model of horizontal expansion was confined to the two products case.⁶ The effect of competition does not take into account the total number of firms in the market and the overall effects on them. The more complicated model of a firm selling different product groups, and the interactions therein has not been analyzed. The model of vertical integration is developed for the particular instance of backward integration only. Input substitution possibility is ignored in the analysis. The reaction of rival firms and the general impact upon the market of such integration has not been examined. It is hoped that extensions of the framework developed in this study, to overcome these drawbacks, would provide an interesting agenda for the future.

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⁶Each product group examined consisted of two products each. This however did not preclude the generality of the insights that could be obtained.

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NATURE OF THE PROBLEM

1.1 The Issues Involved

Traditionally, the firm is viewed as a technical unit which converts scarce resources into goods and/or services for the consumers. Much of microeconomic theory is concerned with the nature of the price and output decisions of a firm. The main emphasis in such studies is the effect of market structure on the performance of the firm. Certain distinguishing features of the market are taken to be the only factors determining the efficiency of resource allocation. Thus, though the firm is regarded as an institution which influences resource allocation, it is considered to be more of a 'black-box'. Very few attempts have been made to study its internal structure.

The rapid growth of firms into large and diversified companies created difficulties for justifying the theory of the single-product-single-ownership firm as being useful in providing insights into the behaviour of firms in reality. Basically, there are three important aspects of a firm in the modern world:

- (i) Observed diversity of products/services that the modern firm offers.
- (ii) Structurally complex modes of production and organization of resources within the firm.

(iii) The existence of significant costs of using the market mechanism.

The first aspect is the multiproduct nature of almost all firms. Many of them offer a great many varieties of similar products and/or a host of seemingly unrelated products. Such firms are said to be horizontally or conglomerately integrated. In many instances firms are also seen to be vertically integrated though the degree of integration may vary from one firm to another.

The second aspect would include not only the technology of production, but also the entire edifice of managing, organizing and monitoring of production, finances and marketing operations.

The third aspect essentially refers to the concept of transaction costs that have a significant bearing whenever business activities are undertaken (Coase 1937; Williamson 1971, 1984). It would seem reasonable to expect that a theory of the firm would reflect these aspects at least in a partial way.

1.2 Some Initial Analytical Attempts

To a limited extent, traditional theory sought to incorporate some of these features. Specifically, the recognition of product diversity as an undeniable part of reality led to the theory of Monopolistic Competition (Chamberlin 1948) and much more recently to considerable literature devoted to the modelling of product diversity (Lancaster 1975; Spence 1976a, 1976b; Dixit and Stiglitz 1977; Hart 1979).

However, the significant feature of multiple output production by a firm is not embodied in these studies. On the other hand, multiproduct characterization can be found in many works starting from a simple treatment by Hicks (1946) to a more sophisticated examination of the production and cost functions (Fuss and McFadden 1978; Laitinen 1980; Baumol, Panzar and Willig 1982). Joint production is considered as one of the main features of multiproduct production and therefore the neoclassical production and cost functions were extended to take this into account (Pfouts 1961; Hirota and Kuga 1971; Burmeister and Turnovsky 1971; Hall 1973). With the exception of Baumol et al. (1982) much of the work is strictly an extension of the neoclassical framework and provides very few insights into the basic reasons for the emergence of multiproduct firms. Also, the question of product diversity was taken up in the restricted sense that none of the products were commercially related to the others. Similarly, cost interrelationships were explicitly considered by Baumol et al. (1982). However, their main emphasis was on determining efficient industry configuration rather than theorizing at the firm level. A similar recognition of multiproduct features are found in Sharkey (1982) who deals with the conditions for a natural monopoly rather than the questions of why firms are large and diversified in the first place. Both the above mentioned studies have, to some extent, recognized that the characterization of production activities in a firm cannot be complete unless the nature of the technological and

organizational modes are taken into account. Recent attempts to characterize product diversification in a multiproduct setting can be found in the works of Nicolaou and Spencer (1975), Mussa and Rosen (1978), Itoh (1983) and Katz (1984). However, these models have ignored the possibilities of any cost advantages that would be inherent in multiproduct production. Indeed, there are persuasive reasons to believe that the existence of economies of producing several outputs jointly would furnish a motive for firms to become multiproduct enterprises.

Parallel to the above mentioned developments was the recognition that large and diversified firms can no longer be coordinated by a single owner. Instead, they present a bewildering variety of organizational forms that are both large and complex in nature. The organizational dimension of the firm, a recognition and examination of which emerged as an important step in doing away with the 'black box' concept, has chosen to view the firm as a collection of resources rather than a technical unit which always functions 'most efficiently'. Specifically, the separation of ownership and management in large companies was recognized and the possible conflicts in the objectives which might result from this were examined (Berle and Means 1967). Also, the neoclassical assumption of profit maximization as the motivating goal was replaced with alternative goals which are likely to be pursued by the firm.

Perhaps the most important and significant starting point for the work along the organizational dimension was that of Penrose (1959) wherein the firm was viewed as a collection of resources. An important feature which makes for multiproduct

production in the Penrosian framework is the emergence of managerial excess capacity over time. Thus, in contrast to neoclassical theory wherein the emergence of "excess capacity" was attributed to the market structure (with excess capacity being defined in the purely technical fashion of not operating on the minimum point of the average cost curve) the Penrosian framework views the availability of excess capacity as emerging entirely from the specific training and learning by doing within the firm over time. Note that this definition of excess capacity is qualitatively different from the neoclassical definition. For, in contrast to the existing literature, it focusses upon a particular organizational dimension within the firm. The absence of a market for the excess capacity so generated would motivate the firm into expanding into the same product line or into diversifying its activities by using up this excess capacity.

More recently Baumol et al.(1982) recognized the possibility that cost-savings may result from the simultaneous production of several products in a single enterprise in contrast to their production in isolation. Such economies of 'scope' arise largely because "different products require much the same overheads" (Hicks 1935, p. 372, quoted in Baumol et al. 1982, p. 77). It is clear that such common overheads would include organizational features in addition to technological relatedness in production. A similar concept is that of 'firm subadditivity' by Sharkey (1982) which refers to the economies obtained by the ability to coordinate production activities within a single firm more efficiently than by

two or more competing firms.

The above arguments show that increased attention has been paid to the possible role that the organizational variables play within the firm which influence its decisions and outcomes. While organizational overheads may create excess capacity within the firm over time, they may also contribute to the existence of economies of joint production. While the former contributes to economies of scope arguments, the latter would provide the basic motivational factor which makes for diversification. However, for analytical purposes, the literature offers very little in terms of a formal model of the theory of the firm incorporating the above features as a means of exhibiting diversification activities. For, as mentioned earlier, both Baumol et al. (1982) and Sharkey (1982) are concerned with issues of industry structure and natural monopoly respectively.

The famous article by Coase (1937), on the nature of the firm, emphasized the presence of market transaction costs and attributed the very existence of a firm to the more efficient (less costly) way of conducting and organising production vis-a-vis the market. Williamson (1975, 1979), on the other hand, recognized that the same factors that make transactions costly in a market also tend to operate within an internal organization. Important as it seems, the transaction cost arguments have yet to be incorporated into the mainstream of economic analysis. A multiproduct production mode that is particularly amenable to the transaction cost arguments is that of a vertically integrated firm. Williamson (1971, 1984)

is explicit in his recognition of the role of transaction costs and market imperfections in vertical integration. However, most of the models on vertical integration have ignored transaction costs from the set of key variables (Dixit 1983; Perry 1978; Carlton 1979). A detailed description of the nature of transaction costs is deferred to a later chapter.

1.3 The Need for Further Study

The above perusal of the existing literature serves to highlight the following points:

(i) Whenever firms consider diversification of their activities, there are reasons to believe that diversification can be through internal expansion with certain inherent cost advantages that accrue to the firm in question. At a more analytical level, it becomes apparent that any characterization of the multiproduct production and cost functions would be expected to exhibit significant interrelationships between the products produced. The literature on this aspect is rather inadequate as far as firm level characterizations are concerned.

(ii) The case of demand interrelationships has been given much more importance than the cost aspects.

(iii) The transaction cost arguments have not yet been embodied into the main stream of micro-economic analysis although they have a significant role in the theory of the firm.

In the present study an attempt is made to incorporate the possible effects that particular structural modes of production and organization and the existence of market transaction costs exert on the decisions of the firms to be diversified

horizontally or be vertically integrated. Specifically, the inherent cost advantage in multiproduct production has been characterized through the concept of weak cost complementarity and significant interrelationships in the cost as well as demand structures have been incorporated to arrive at decisions regarding the efficient quantities of the output vector.

In examining the model of horizontal integration both the possibilities of the firm going in for the production of substitute products or complementary products is taken into account. It will be shown that many of the results in the literature dealing with the problem of product diversity with any one firm producing a single product carry over to the case of multiproduct production. At the same time new insights can be obtained about the efficient use of the economies of scope and excess capacities that make for diversification through internal expansion.

Similarly the model of vertical integration presented in this study suggests that the presence of high market transaction costs and market imperfections in the purchase of inputs could also generate economies of scope even if there were no inherent economies of joint production. Thus conceptually horizontal diversification and vertical integration are treated as two distinct forms in which multiproduct production can be organized within the firm. While the former is viewed as emerging due to inherent economies of joint production, the latter is viewed as emerging because of high transaction costs and market imperfections.

The emphasis of the present study, however, is on developing notions of the efficient levels of output given the above mentioned features of multiproduct production. Simple quantity models are constructed and the important questions regarding efficiency of the outcomes of the decisions to either diversify or integrated vertically are unambiguously defined. Sources of possible deviations from efficient levels are identified.

1.4 Organization of the Thesis

The rest of the thesis is organized in the following way. In Chapter Two the basic notion of efficiency that exists in the literature is expounded and its relevance to the present study highlighted. Chapter Three is devoted to the characterization of costs and demand of firms that are horizontally integrated and an a priori notion of cost efficient choices of outputs is suggested. Based upon the hypothesized behaviour of costs and demand, Chapter Four is devoted to deriving the economically efficient choices of output levels. These levels were then compared with the profit maximizing choices against the background of the cost efficient choices of output levels within the firm developed in Chapter Three. Chapter Five extends the analysis on horizontal integration by considering the effect of entry as increasing competition for at least one of its products. Particular attention is paid to the effect of outside competition on a conglomerately integrated firm and one that sells complementary products. This chapter, along with the

previous two chapters, consists of the analysis of multi-product output decisions when the outputs are marketable.

Chapter Six deals with the vertical integration aspect of multiproduct production. Again, the notion of efficiency under vertical integration is developed using the transaction cost arguments. This is also seen to offer economies of scope and, in some cases, cost complementarities in vertically integrating the firm's activities.

Chapter Seven summarizes the results obtained in the thesis.

EFFICIENCY : THE ISSUES

2.1 The Basic Notion of Economic Efficiency

The concept of efficiency in economic theory concentrates on two aspects: (i) Efficiency relating to the firm, and (ii) Efficiency related to the economy as a whole, i.e., producers and consumers. Economic theory concerns itself with an evaluation of the efficient configurations of output and prices within different market structures.

The efficiency concept related to the firm is often called technical efficiency and refers to the firm operating on the boundary of the production possibility set. Duality between production and costs initiated by Shephard (1953) established that a technically efficient firm would also be operating on its minimum-cost expansion path. However, much of the subject matter on efficiency notions focus upon the fulfilment or otherwise, of the more general conditions for the maximization of total welfare or surplus. When efficiency of the firm is treated in the context of the welfare of both producers and consumers, the conditions of technical efficiency within the firm are assumed to hold. That is, the firm is expected to work with the minimum possible costs of producing specific output levels. This is a logical necessity if the demand conditions are exogenous to the decision process of the firm. For, then, the maximization of surplus would amount to minimization of costs. The conclusion

regarding efficient pricing is the familiar Price = Marginal cost principle ($P = MC$ from now on). This rule determines what is generally known as the "economically efficient" level of output.

2.2 Efficiency and Product Diversity

The recognition of product diversity as a fundamental feature of the monopolistically competitive market raises the question of the optimal number of products in addition to the efficient quantities of these. The welfare implications of product diversity have been examined in Spence (1976a,b) and generalized by Dixit and Stiglitz (1977). In both these studies optimal product diversity is obtained by maximizing a benefit function (defined over the quantities of an arbitrary number of differentiated products) net of industry costs. Spence (1976a) has particularly interesting observations on the reasons for bias in product selection by firms. Product selection is seen to be dependent upon how much of the consumer surplus can be converted into revenue for the firm and consequently its profits. Put differently, the revenues accruing to a firm are the main determinants of whether the products would be produced or not. Since revenues do not necessarily reflect consumer surplus on a one-to-one basis there is reason to suspect that some products which contribute more to consumer surplus than to revenues may not be produced at all.

While this yields useful insights the analysis assumes that each firm produces only one variant of the industry's

product. The optimal number of firms in the industry also determine optimal number of products in the framework of Spence.

2.3 Efficiency, Integration and the Multiproduct Firm

In practice, most firms produce a variety of products. They exhibit interrelationships in demand as well as costs. This recognition of the inherent demand interrelationships renders the measure of consumer surplus ambiguous. Since maximization of welfare and the concept of economic efficiency assumes a theoretically workable notion of consumer surplus, theory has allowed the extension of the economic efficiency rule $P = MC$ to every one of the products assuming that the 'integrability conditions' are generally satisfied. Modelling multiproduct firms on the assumption that they produce goods of different 'qualities' has been a preoccupation of many theorists. Prominent among such studies are the papers by Hagen and Dreze (1978), Mussa and Rosen (1978) and Itoh (1983). Hagen and Dreze show that the benefits from several firms simultaneously producing new products may exceed the sum of the benefits from any one firm doing it alone. Mussa and Rosen aimed at showing the contrast in the price schedules for a monopolistic producer supplying infinitely many related commodities with that of the competitive price schedule. Itoh (1983), following Mussa and Rosen, modelled quality as a one dimensional hedonic attribute and sought to analyse the changes in welfare when a new good is introduced. Overall welfare impacts of a new good were seen to be

sensitive to the ratio of the density of consumers purchasing the goods of higher quality to that of the marginal consumers who switch goods.

The above discussion serves to highlight the important notion that the efficiency concepts, when product diversity and the multiproduct nature of the firm is accounted for, center around the notions of total surplus maximization and that the $P = MC$ rule, which defines the economically efficient choices of output and prices can be extended to the more general nature of the multiproduct enterprise. However, recall from Chapter One that multiproduct production is often a form of joint production and may therefore exhibit significant interrelationships between products, in the production process itself, quite apart from the interrelationships in the demands for them. This would, in turn, have a significant bearing on the cost structure and hence on the total surplus that is to be maximized in the determining the economically efficient choices of output levels. It has been argued in Chapter One that the presence of both organizational as well as technological economies of joint production offer clear advantages of multiproduct production since there would be a reduction in costs by producing several outputs in a single firm vis-a-vis their production separately in specialized firms. Viewed from a more general vantage point, there seem to be at least two alternative arguments for and against integrated firms to be sources of inefficiency. Kahn (1971) has summed up the possible efficiency gains of integrated firms as being cost saving, as the above

arguments suggest, and as being conducive to a kind of competition. The mere ability of firms to integrate and lower costs is regarded by Kahn as some sort of competition to keep other firms "on their toes" (Kahn 1971, p. 257). According to him the market structure will not change adversely since the firm which enters a new business activity by setting up its own facility increases competition in that market. Furthermore, whenever goods and services can be provided by alternative methods, it would be in the interest of the firm to choose a combination of methods in each case which would minimize costs.

Contrary to these notions, Scherer (1980) examines the hypothesis that conglomerate size is specially conducive to predatory pricing, spheres of influence agreements among gaint rivals and non-maximization of profits. It is maintained that financially powerful conglomerates have a greater ability and willingness to engage in sustained price cutting with the intent of disciplining smaller competitors and/or driving them out of business. It has also been argued that the very nature of a multiproduct firm enables it to subsidize its predatory operations with profits from other lines of activity until predation creates conditions that will repay the original subsidy.

Similarly, the arguments of Kahn (1971) and Scherer (1980) and those of the Chicago school in general, represent two differing opinions about the welfare impacts of a vertically integrated firm.

The basic arguments of Kahn (1971) which link efficiency and integration is the postulate that integration works against effective competition. That is, integration is viewed as a source of heightened monopoly power. Kahn views the case for and against vertical integration in terms of a trade-off between ". . . considerations of efficiency on the one hand and purity of rivalry on the other. In the presence of economies of integration (as of scale), the balancing has to be between permitting firms large and integrated enough to enjoy these economies and firms numerous enough and with sufficient opportunity for effective rivalry" (Kahn 1971, p. 255).

A more serious consequence of integration on efficiency has been argued by Scherer (1980). A non-integrated firm cannot hope to compete with its integrated counterparts since they hold strong bargaining positions in both the upstream input markets and the downstream fabrication markets. Moreover, the supply of inputs would be an uncertain proposition for the non-integrated firms. This in itself would be a barrier to entry so that potential entrants may be compelled to enter only on a fully integrated scale. However, the large capital investments required for such an undertaking is not within the reach of all potential entrants. Hence, the incumbents can hold prices above marginal costs by a higher margin before attracting entry.

Closely related to this is the 'price squeeze' argument, wherein an integrated firm may raise input prices while holding prices at the next stage (downstream) constant, thus

subjecting the non-integrated rivals to a squeeze.

The Chicago school arguments on the other hand perceive vertical integration as enhancing efficiency. In particular, vertical integration is viewed as a way of dissolving bilateral monopoly bargaining stalemates and eliminating double marginalization by vertical chain monopolies, and minimizing input distortions. For a special class of cases, they argue, it also leads to lower end product prices.

2.4 Conclusion

The two critical notions that play an important part in all of these arguments relating to efficiency are the cost advantages that are possible through integration/diversification and the emergence of monopoly power since diversified firms tend to be large and may have a tendency to reduce the number of firms in the market.

In the present study, since the emphasis is on developing notions of efficiency at the firm level, attention is paid to the costs aspect of the problem. Specifically whenever the marginal cost of producing a given output is no longer independent of other outputs that the firm produces, the welfare maximizing rule, $P = MC$ becomes meaningful only if the marginal cost curve in question is the minimum possible given the levels of the other outputs.

In addition, it is reasonable to suppose that the very nature of the products that the firm envisages to sell will, through demand, affect the firm's endeavour to offer the products at their lowest possible marginal costs.

The more general questions of efficiency in a market are not pursued except in a limited way. However, the more relevant questions of how efficiency within the firm is affected via entry of an another firm into the same market is analysed. In the rest of the thesis, the terms 'welfare maximizing', 'total surplus maximizing' and 'economically efficient' are treated as synonymous and are used interchangeably.

CHAPTER 3

HORIZONTAL INTEGRATION : MAIN ASPECTS

In this chapter, an outline of the pertinent features of horizontal integration which will be incorporated in the model are presented. Specifically, the properties of costs and demand will be developed keeping in view the features peculiar to firms undertaking horizontal integration. The theoretical structuring of the problem will be confined to the two variable case to facilitate exposition.

3.1 The Characterization of Horizontal Integration

Consider a firm which is initially producing only one product in a monopolistic market. For various reasons the firm may envisage an expansion by integrating into the production of other products. When these other products are related commercially to the existing product the firm is considered to be horizontally integrated.

Integration into other product lines can be achieved through diverse organizational mechanisms. Prominent among these are:

(i) The firm currently producing a product Y_i enacts a merger with another firm producing Y_j and lends its brand name to this product. This would be a case of Horizontal Merger.

(ii) The firm acquires another firm which is producing Y_j .

(iii) The firm undertakes production of Y_j through internal expansion.

The literature on Horizontal integration concentrates on the first two mechanisms. Very little attention has been paid to the characterization of the internal expansion mechanism. However, as has been noted in Chapter One, the development of the theory of the multiproduct firm, together with an increasing awareness that the firm is a collection of resources in the broadest sense of the term provides an opportunity to obtain a better understanding of horizontal integration through internal expansion. It had been argued, in Chapter One, that the literature offers many reasons to believe that the very existence of multiproduct firms is due to economies of producing an output range jointly. At a more specific level, a basic reason why firms would produce a product line would be the presence of certain variable inputs which once acquired can be used in producing two or more products. Excess capacity at the plant level may add to the excess capacity so generated in these specialized inputs which would then be used to produce another product. Apart from indivisibilities in the technological process of production itself, one can think of financial, marketing and managerial inputs as being variable and capable of being applied equally well to more than one product. Such inputs are called 'Public inputs' (Baumol et al. 1982). Similarly the cost advantages of a firm's ability to coordinate, manage and monitor the production and distribution of a vector of related products are also results of the more efficient use of certain inputs

even a line of products, but rather its capacity to produce."

Whereas the Penrosian argument emphasizes managerial excess capacity, it would seem equally valid when applied to areas like finance, marketing and so on.

Thus, given the emergence of excess capacity either from a technological or an organizational viewpoint, and the fact that there are economies to be derived from producing a range of products due to the presence of public inputs, it would seem reasonable to characterize horizontal integration by internal expansion as one which exhibits economies of joint production. Note that this does not mean the production of joint products. It is also important to observe that during any given period of time these economies would be finite and limited. The firm's ability or potential to diversify using resources that are already at hand will be necessarily limited by the capacity to use the variable inputs into diversification activities. This may be further restricted if the markets for the said inputs are not competitive, or there is very little idle capacity left with the firm in their usage. Ofcourse, it can always be argued that the firm may go in for an expansion of all inputs concerned, but this would be a realistic long term phenomena and the question of inherent cost advantages in such expansion activities would depend upon many factors that have little bearing while focussing upon the internal allocation of variable resources within the firm.

Hall (1973) specified the technology with several kinds of output as being joint or non-joint depending upon the

existence or otherwise of economies/diseconomies of joint production, using the principles of duality (Shephard 1953; Uzawa 1964; McFadden 1978). The joint cost function was derived for joint technology specifications as well as for non-joint technologies. It is now well known that if the technology is non-joint, then the associated cost function is strongly additive; otherwise there will be significant cost interrelationships.

Economies of joint production almost always means that there is a cost advantage in producing the set of products together as opposed to producing them separately. This property of joint production is said to exhibit what is known as Economies of Scope. It is clear that the sources of economies of scope are found in the characterization of internal expansion alluded to above; whether it be the concept of 'public-input' by Baumol et al. (1982), or the emergence of managerial excess capacity (Penrose 1959), or the concept of firm subadditivity and 'plant' subadditivity (Sharkey 1982) which indicate the presence of unavoidable externalities in the production process itself. In fact there is every reason to believe that all of these factors contribute to make for internal expansion of firms. Thus sufficient conditions for generating a motivation on the part of the firm to horizontally integrate would be jointness in the underlying technology as well as the existence of the conditions under which economies of scope emerge. Usually it is the associated cost function rather than the production function that is used to exhibit the above

characteristics. Therefore, an examination of the nature of the cost function becomes relevant at this stage.

3.2 The Nature of the Cost Function

Among the multiproduct cost concepts available in the literature, the concept of weak cost complementarity can be shown to be sufficient to indicate the presence of jointness in production and economies of scope.

Formally, the concepts are defined below.

Definition 1. Economies of Scope

Economies of scope are said to exist whenever it is cheaper to produce several outputs in a single enterprise as compared to producing each of them in isolation; i.e., each by its own specialized firm.

Let $C(Y_1, Y_2)$ represent the total cost function of producing two products whose quantities are represented by Y_1 and Y_2 . Then, economies of scope are said to exist if:

$$C(Y_1, Y_2) < C(Y_1, 0) + C(0, Y_2)$$

Definition 2. Weak Cost Complementarity

Weak cost complementarity requires that the marginal or incremental costs of producing any output decline when any other output is increased. For the two product case, weak cost complementarity implies,

$$\partial^2 C(Y_1, Y_2) / \partial Y_1 \partial Y_2 \equiv C_{12}(Y_1, Y_2) < 0$$

for a certain range of output values. It is usual to assume that the total cost function $C(Y_1, Y_2)$ is continuously

differentiable with continuous first and second order partial derivatives. Hence,

$$\begin{aligned}\partial^2 C(Y_1, Y_2) / \partial Y_1 \partial Y_2 &\equiv C_{12}(Y_1, Y_2) \equiv \partial^2 C(Y_1, Y_2) / \partial Y_2 \partial Y_1 \\ &\equiv C_{21}(Y_1, Y_2)\end{aligned}$$

Notice that if the second cross-partial derivative of the cost function is non-zero, the cost function is no longer additive and hence the production is not non-joint. Also, by the definition of cost complementarity, there are clearly cost advantages to the firm in the relevant output range which imply economies of joint production in that output range.

Next it can be shown that cost complementarity in production is a sufficient condition for the presence of economies of scope. The general proof is found in Baumol et al. (1982). The proof here has been confined to two variables only. However, the insights obtained by modelling horizontal integration with only two products are still sufficiently general to exhibit a large number of possibilities as will be seen later.

Lemma 1. Cost complementarity is a sufficient condition for economies of scope.

Proof:

If $C_{12}(Y_1, Y_2) < 0$, for all $\bar{e} < Y_1$,

$e < Y_2$, then,

$$\begin{aligned}
0 &> \int_0^{Y_2} \int_0^{Y_1} \{c_{12}(\cdot, \cdot) d\bar{\theta}\} d\theta \\
&= \int_0^{Y_2} \{c_2(Y_1, \theta) - c_2(0, \theta)\} d\theta \\
&= C(Y_1, Y_2) - C(Y_1, 0) - (C(0, Y_2) - C(0, 0))
\end{aligned}$$

Thus since $C(0, 0) \geq 0$

Therefore, $C(Y_1, Y_2) < C(Y_1, 0) + C(0, Y_2)$, which is the definition of Economies of Scope.

Q.E.D.

Notice that weak cost complementarity places no restriction on the multiproduct cost function. Further, cost complementarity is a global concept that must be verified for an infinite number of inequalities such as the one in definition 2.

More important is the idea that cost complementarity need not hold for the entire output range that is feasible. Sharkey (1982) cites two instances wherein this happens:

(i) The presence of product specific fixed costs, and

(ii) If capacity constraints in the use of some common inputs to produce two or more products become binding.

The last point has been argued already. This observation makes for the idea that presence of excess capacity in the common inputs is necessary for the entire arguments in favour of internal expansion. Baumol et al. (1982) acknowledge this when they argue that cost advantages may accrue to the firm if there is no "congestion" in the use of these common inputs. Also recall that the Penrosian arguments view several organizational features as common inputs which, once acquired by the firm to produce an output, can be used (costlessly) for the production of other products as well. The emergence of excess capacity in these was viewed as one of the main reasons for diversification.

Thus, at any given time, the sum total of these organizational as well as technological features that serve the purposes of common inputs would constitute a part of the variable factors of production.¹ Given this nature of the factors of production, excess capacity in them would be necessary for cost complementarities to emerge and at the

¹Perhaps it was this recognition that inputs which would otherwise have been labelled as 'fixed' had there been a single product, now become a separate class of "variable factors" in the multiproduct set up, that led Ferguson (1969) to develop his cost function. However, since he assumed that there exist costs of "switching" from one line of products to another, his production function did not exhibit any economies of joint production, since the switching costs can be looked upon as product specific fixed costs in the Baumol-Sharkey framework.

capacity limits (capacity cannot be defined in the neoclassical way when organizational features are also taken into account) cost complementarities are expected to be exhausted.

Much depends upon the amount of capacity utilization of the fixed and variable factors for existing product/s, since in the short-run, the excess capacity generated would provide incentives for expansion.

3.3 Some Observations on the Behaviour of Costs

Consider, at a more analytical level, the case wherein a firm is producing an initial product Y_i . If this value of the output is such that there is no excess capacity, and hence no scope left for an alternative use of the firm's resources without decreasing the production of Y_i , then there are no cost complementarities in production and no economies of scope exist for internal expansion into other products. However, whenever there are such economies of scope, there would be a cost advantage in expanding into Y_j (another product). Using up of this kind of excess capacity in the public inputs would amount to a more efficient total variable cost configuration than before and hence the following notion of cost efficient levels of integration would emerge.

Consider an initial value of $Y_1 (= Y_1^0)$ that the firm is producing. Assuming that there exists excess capacity in the utilization of the common inputs and that a product Y_2 could be produced by utilizing this, it is evident that,

initially cost complementarities (and hence economies of scope) would exist with respect to Y_2 , and, specifically for any given level of Y_1 , there would exist some level of Y_2 ($= Y_2^0$) at which such cost advantages would be exhausted. The point A in Figure 3.3.0 is representative of this. For larger initial values of Y_1^0 it is expected that the cost efficient choices of Y_2 (those values that exhaust cost complementarity) would be smaller since larger the initial production level the less the scope for diversifying through internal expansion alone and hence the lower would be the bounds on the level of Y_2 . Theoretically, for a given size of a firm, which includes both the technological as well as the organizational apparatus, there exists a locus of Y_1 - Y_2 combinations which would be cost efficient. Let Y_1^0 represent a level of Y_1 which leaves no economies of scope nor any cost advantages for expansion into Y_2 . Then $Y_2 = 0$. For every initial level of $Y_1^0 < Y_1^0$ there exists a Y_2^0 such that,

$$C_{12}(Y_1^0, Y_2^0) \equiv C_{21}(Y_1^0, Y_2^0) = 0$$

which means that the marginal cost of producing Y_1^0 would be a minimum if the level of Y_2 is Y_2^0 . This also means that, given a marginal cost of producing Y_1^0 with $Y_2 = 0$ the presence of economies of joint production in the use of common inputs are reflected in the fact that up to $Y_2 = Y_2^0$, the marginal cost of Y_1^0 is monotonically decreasing over the entire range of 0 - Y_2^0 as Y_2 is expanded in production.

However, the value of $Y_2 = Y_2^0$ would represent the range of Y_2 (given $Y_1 = Y_1^0$) for which this property of cost

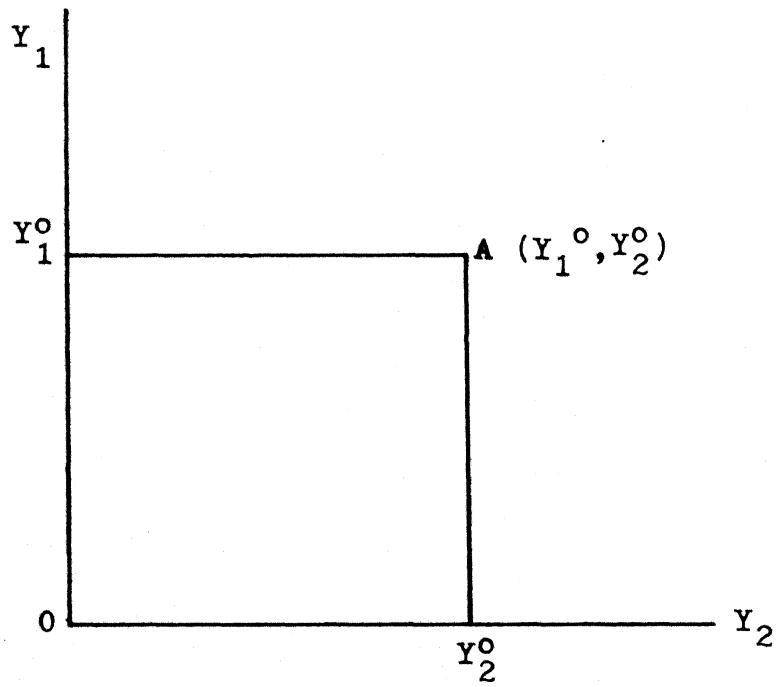
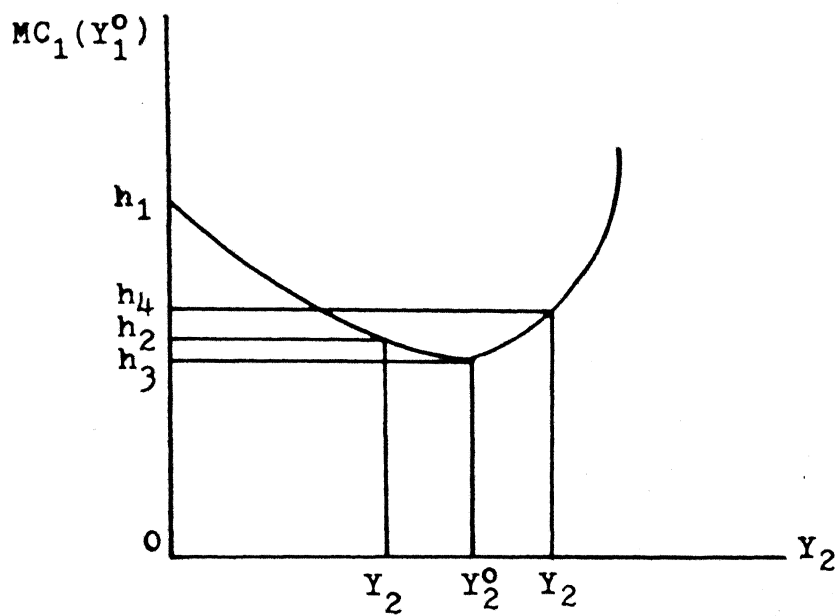


Figure 3.3.0.

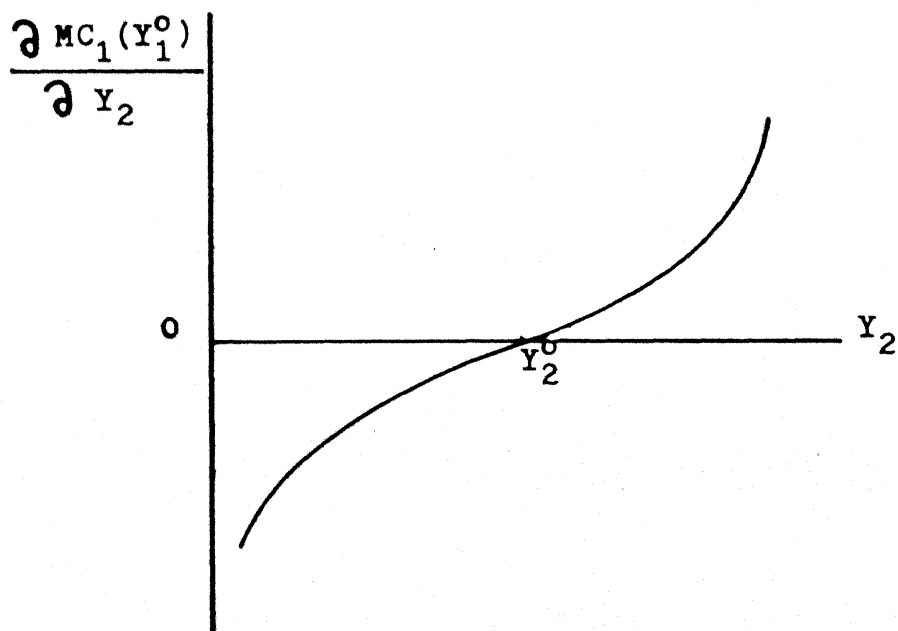
complementarity holds. If Y_2 is to be expanded beyond Y_2^0 then congestion effects in the use of common inputs, capacity constraints, and the need for product specific fixed cost investment will add both to total fixed costs as well as variable costs, and cost complementarity would fail to hold. Indeed, a steep increase in the marginal costs of production for both Y_1 and Y_2 is the more likely consequence.

From the preceding arguments the assumed behaviour of marginal cost of Y_1 with respect to changes in output of Y_2 is depicted in Figure 3.3.1(a). The asymmetrical u-shaped curve which depicts the changes in the marginal cost of producing a given level of $Y_1 = Y_1^0$ must be interpreted as a downward shift in the marginal cost curve for Y_1 , as shown in Figure 3.3.1(c). It will be shown that what is true for Y_1^0 , would also be true for all values of $Y_1 < Y_1^0$ and hence the shifts in the marginal costs would be monotonic in the relevant range (i.e. $0-Y_1^0$ and $0-Y_2^0$). The asymmetry in the changes in the value of the marginal cost of Y_1^0 , when Y_2 is increased beyond the cost efficient quantity Y_2^0 , reflects the idea that beyond the scope of the enterprise it is assumed that costs rise steeply in keeping with the preceding arguments.

The locus of all such Y_1 - Y_2 combinations for which cost complementarities are exhausted would constitute cost efficient choices, and hence is called the Efficient-boundary or simply the E-boundary. Although the exact shape of this boundary cannot be determined a priori, it is clear that it would be negatively sloped in the Y_1 - Y_2 plane and would

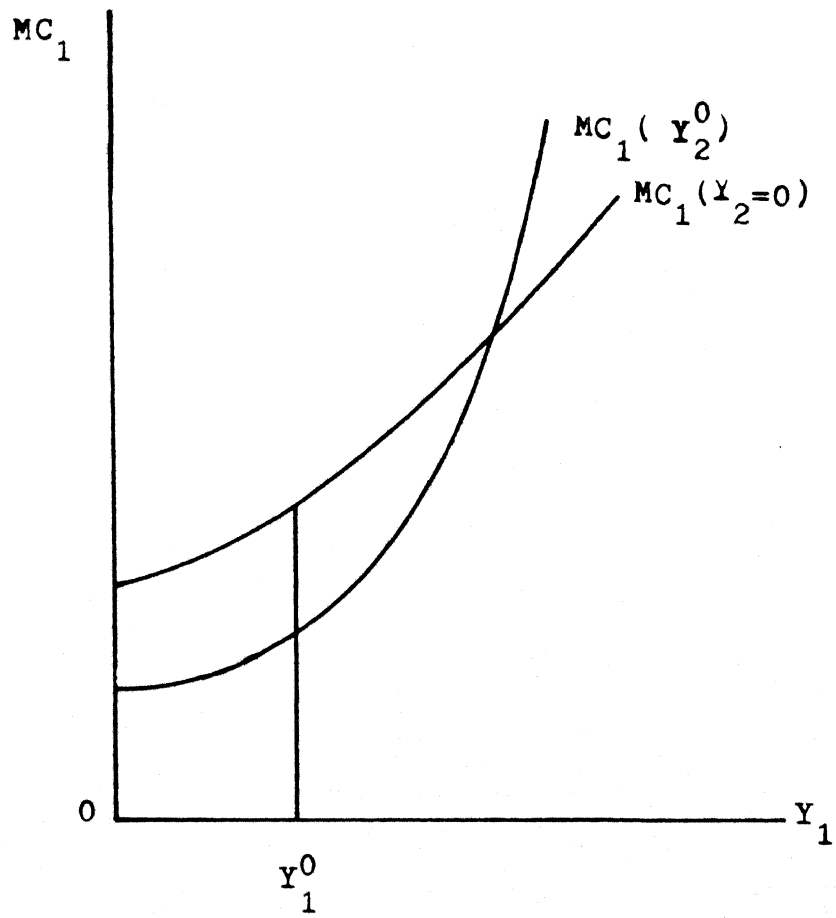


(a)



(b)

Figure 3.3.1



(c)

(Figure 3.3.1.)

extend from one axis to another. Without loss of generality, it is assumed that this boundary is ~~convex~~^{concave} to the origin as portrayed in Figure 3.3.2. The properties of the marginal cost curves can be deduced entirely from the construction of the E-boundary alone and hence it becomes a convenient tool of analysis. These properties are discussed below:

(a) Monotonicity of Marginal Cost Curves:

Let Y_1^0 be an initial value of Y_1 being produced. Then, there exist cost complementarities and hence economies of scope in the production of Y_2 . By construction, this would mean that the marginal cost of Y_1^0 would decrease monotonically as the firm expands into the production of Y_2 up to a point $Y_2 = Y_2^0$. This also means that $MC_1(Y_1^0)$ (marginal cost of Y_1^0) at point A is greater than $MC_1(Y_1^0)$ at point B. Consider any value of $\bar{Y}_1 < Y_1^0$. $MC_1(\bar{Y}_1) < MC_1(Y_1^0)$ when $Y_2 = 0$. However, $MC_1(\bar{Y}_1)$ would also decrease monotonically for increases in Y_2 up to Y_2^0 (and for higher values of Y_2 also). A similar argument holds for any value of Y_1 between $0 - Y_1^0$. Hence the marginal cost curve for Y_1 would shift for changes in Y_2 as shown in Figure 3.3.3. Thus, all points in the interior of the boundary are values of Y_1 and Y_2 for which strict cost complementarity in production holds. In this sense, for any given value of $Y_1 = Y_1^0$, Y_2^0 is the value of Y_2 which minimizes the marginal costs of producing Y_1 . This also minimizes the incremental costs of producing that level of output. Incremental costs would be the total costs associated with producing Y_1^0 level of output as contrasted with not producing it at all.

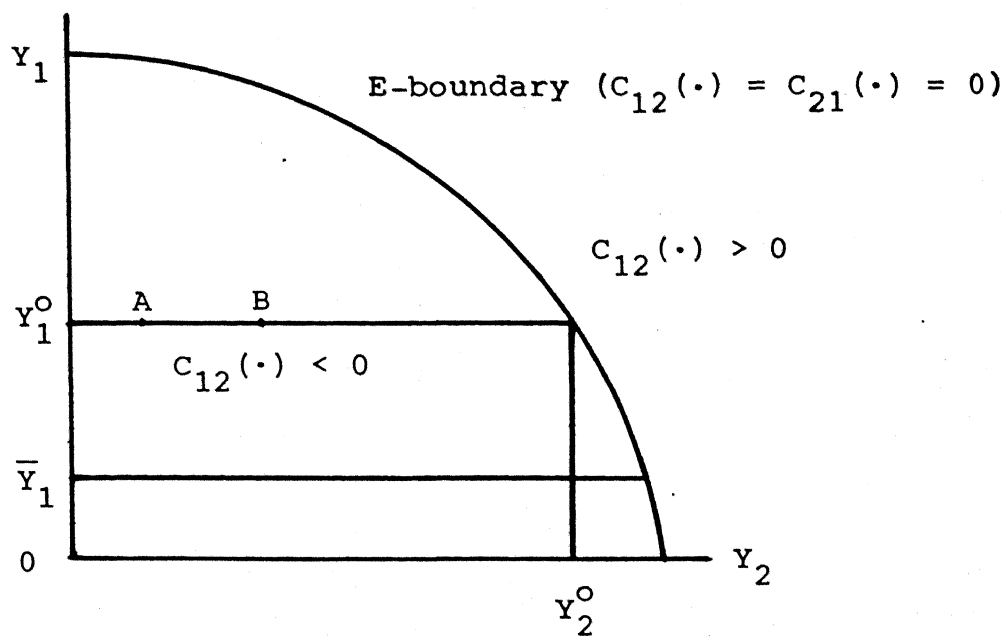


Figure 3.3.2

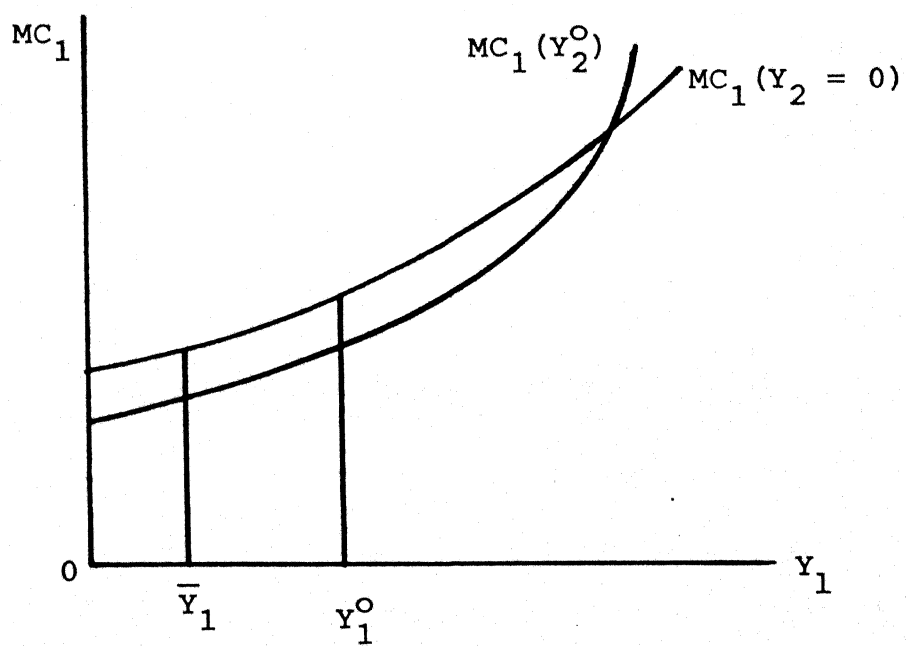


Figure 3.3.3

This is easy to see since, given the monotonicity of the downward shifts in the marginal cost curves in the range of cost complementarity, the area under the marginal cost curve for Y_1^0 has come down with the introduction of Y_2 and reaches a minimum for a value of $Y_2 = Y_2^0$.

(b) Behaviour of Marginal Costs Beyond the E-Boundary

According to the assumptions of steep increases in costs beyond the E-boundary, we would expect the marginal cost curves to shift upwards again, by amounts which would increase the total incremental costs by a large margin. This is shown in Figure 3.3.4. The figure shows that the decrease in the total area under the marginal cost curve (which is abc) would always be less than the area bdf and hence total incremental costs are expected to increase in the region where cost complementarity does not hold.²

²Both Coase (1946) and Bailey (1954) recognized the possibility of cost complementarity and increasing marginal costs whenever two or more products are produced. However, these two cases were not simultaneously incorporated into either of their analysis. Coase did not commit to the notion of monotonic downward shifts in the marginal cost curves and hence on whether total incremental costs were rising or falling when simultaneous production of two or more commodities is undertaken. Bailey (1954, p. 31) was more explicit about the downward shifts in the marginal cost curve when he says ". . . it must be true over a greater part of their (the products) joint cost surface, or else the two commodities would not be produced together" (bracketed term added). However, the reasons why cost complementarity in production can emerge are not stated.

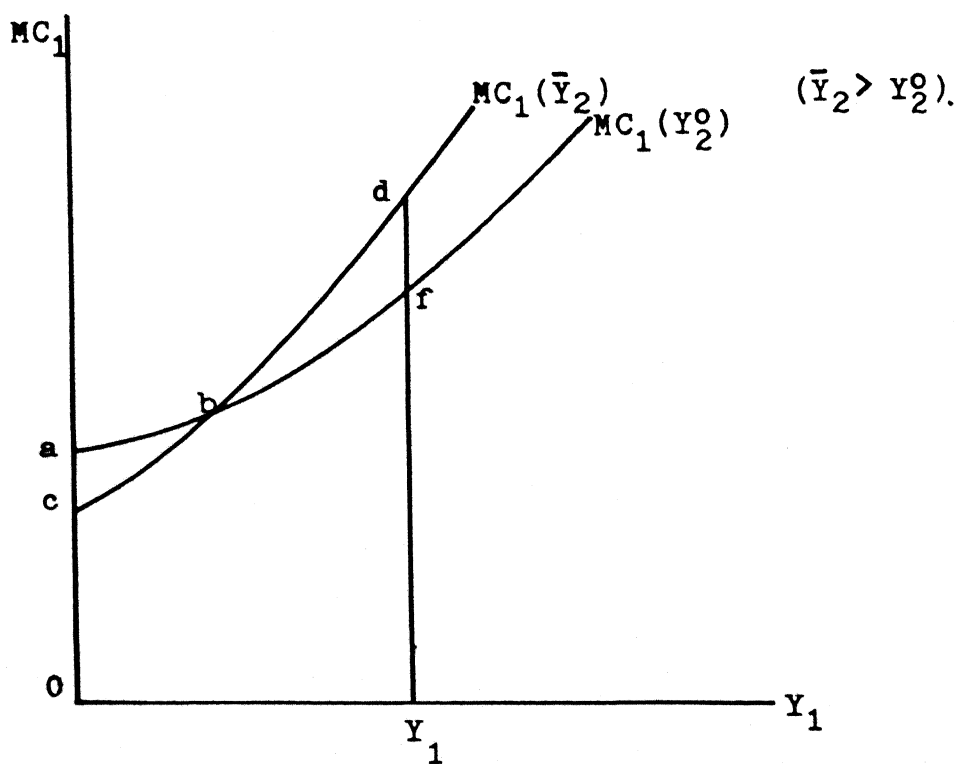


Figure 3.3.4.

Based upon the above description of the behaviour of the cost function the following general properties of costs would be assumed throughout the rest of the analysis. Formally,

- (i) $C(Y_1, Y_2)$ represents the total costs of producing Y_1 and Y_2 quantities of both goods respectively. It is assumed that the fixed costs do not change and are therefore suppressed in the cost function.
- (ii) $C(\cdot)$ is continuously differentiable with continuous First and Second-Order Partial derivatives. Specifically,

A) $\partial C(\cdot)/\partial Y_1 = C_1(\cdot) > 0$

B) $\partial C(\cdot)/\partial Y_2 = C_2(\cdot) > 0$

(positive marginal costs of producing Y_1 and Y_2 respectively.)

C) $\partial^2 C(\cdot)/\partial Y_1^2 = C_{11}(\cdot) > 0$

D) $\partial^2 C(\cdot)/\partial Y_2^2 = C_{22}(\cdot) > 0$

(increasing marginal costs of production.)

E) $\partial^2 C(\cdot)/\partial Y_1 \partial Y_2 \equiv \partial^2 C(\cdot)/\partial Y_2 \partial Y_1 \equiv C_{12}(\cdot) = C_{21}(\cdot)$

(property of continuity.)

- F) $C_{12}(\cdot) \not\leq 0$, so that there exist economies of scope in the form of cost complementarities in the production of Y_1 and Y_2 over some range only. The relevant range and the properties of the marginal cost curves are defined by the E-boundary.

- G) $C(\cdot)$ is convex, though not strictly. Convexity of costs have been assumed elsewhere in the literature, notably by Baumol et al. (1982).

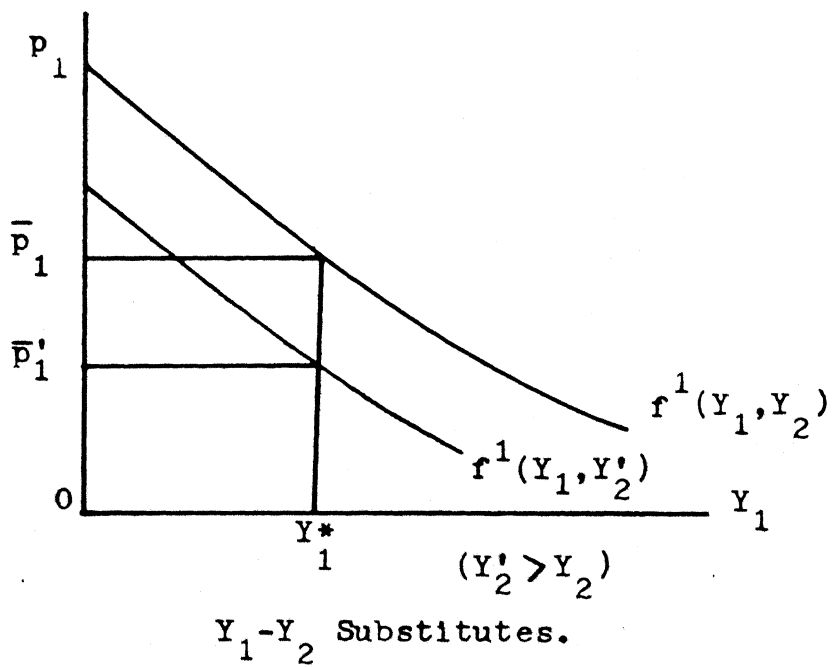
This completes the statement of the assumptions and properties of costs that would be used in the model. The demand aspect of the problem is taken up next.

3.4 The Characterization of Demand

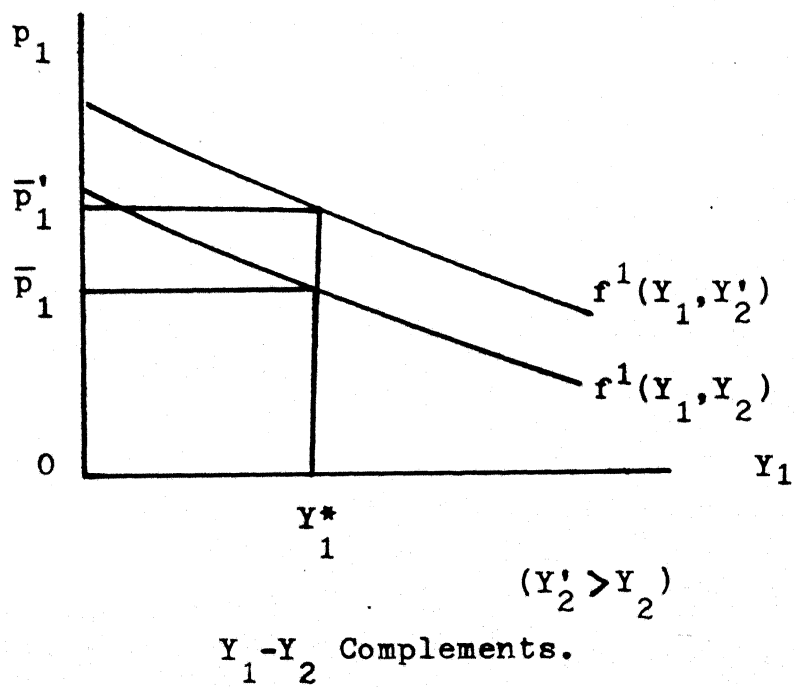
By its very definition, horizontal integration means that the products are commercially related. That is, the demand curves for each product are not independent of the variations in the quantities/prices of the other products. There are two possible ways in which the products may be related. When a decrease in the price of one product decreases the willingness to pay for the other, then the products are substitutable. If it increases the willingness to pay then they are complementary. These two cases are shown in Figure 3.4.0(a,b).

Substitutable products may include those products which form a variant of a particular product 'type'. Although some economists would be of the opinion that this is merely product differentiation, such product 'variety' generates demand functions which show interrelatedness. Also, product variety may, in fact, turn out to be more than simple product differentiation since characteristics like quality, durability and other such features will be the main factors that differentiate it in the eyes of consumers. Demand interrelationships, if and when they exist, must be taken into account in deciding the optimal product mix of a firm since these have a direct bearing upon revenues obtainable.

The recognition of the demand interrelationships is seen in the recent attempts to characterize product diversity by a single firm (see Mussa and Rosen 1978; Itoh 1983;



(a)



(b)

Figure 3.4.0.

Ireland 1983; Katz 1984). The focus of all these studies, as mentioned earlier, was on modelling the 'quality' attribute of a commodity and the examination of the resulting interrelationships therein. Demand interrelationships are usually exhibited in terms of the shifts in the demand curves, as a result of changes in another product's price or quantity. The following properties of demand for substitutable products is illustrative of the way in which demand interrelationships are treated in the literature.

Let the demand for two substitutable products Y_1 and Y_2 be represented by the two inverse demand functions

$$p_1 = f^1(Y_1, Y_2) \quad \text{and} \quad p_2 = f^2(Y_1, Y_2)$$

$\partial p_i / \partial Y_j \leq 0$, $i \neq j$ is the definition of Weak Gross Substitutes. Strict inequality holds for Gross Substitutes. This would represent a downward shift in the demand curve for Y_i when Y_j increases as shown by the previous Figure 3.4.0(a). Spence (1976b) suggests that the downward shift in the demand curve may become larger for larger values of its own product quantity as shown in Figure 3.4.1. The 'steepness' of the inverse demand function, it is argued, can come from two sources:

- (a) The product may have a naturally limited set of buyers with a highly variegated valuation of the product, and
- (b) The presence of competitive products tend to take away the consumers with lower valuations of the product.

On the other hand those studies which model 'quality' as the main differentiating feature, assume that the substitutability is confined to the 'nearest neighbour' if 'quality'

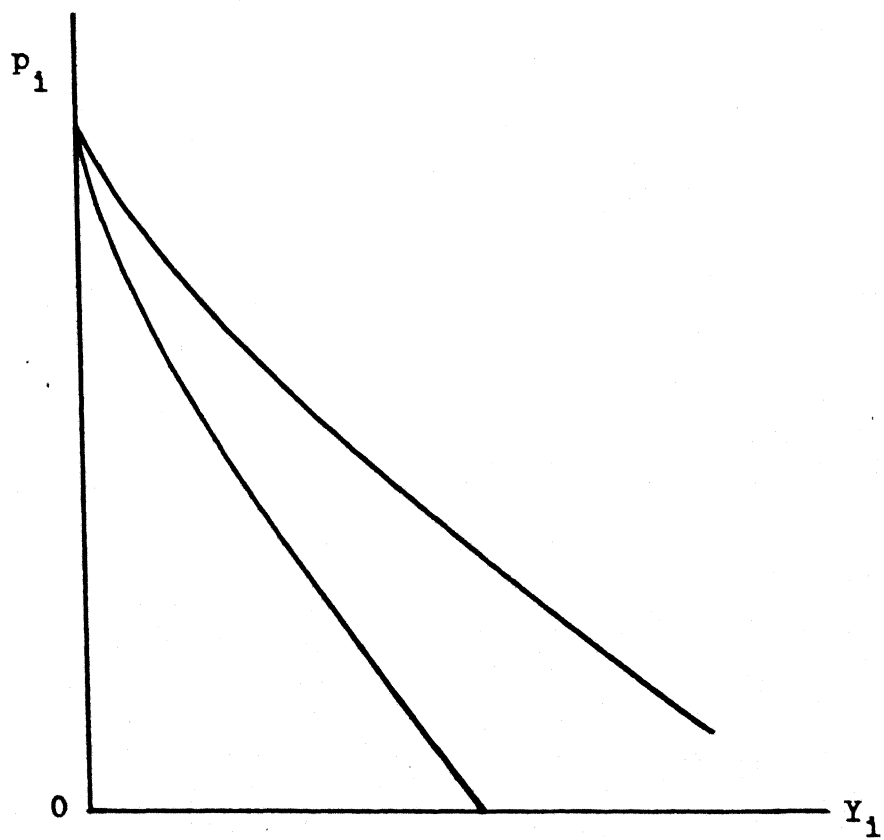


Figure 3.4.1

could be arranged on a simple scale with the highest quality and the lowest quality products at either end. Obviously, a lot depends upon the density of consumers who buy the particular quality types and the marginal consumers who will switch quality choices for changes in prices. Both Katz (1984) and Ireland (1985) introduced demand interrelations through conjectural variations across the firms.

In the present study, the demand interrelationships mentioned above are modelled in a more general way. The following properties of demand are relevant in the two product case:

Let $p_1 = f^1(Y_1, Y_2)$ represent the inverse demand function for product 1 whose quantity is denoted by Y_1 . The demand function $f^1(\cdot)$ is defined and continuous for all $Y_1 \geq 0$ and $Y_2 \geq 0$ such that,

(a) $f^1(\cdot) = 0$ for $Y_1 \geq \tilde{Y}_1$ given that $Y_2 > 0$. The \tilde{Y}_1 itself will be defined for values of $Y_2 \geq 0$, and is therefore a variable. It is usual to assume that (i) \tilde{Y}_1 is a decreasing function of Y_2 whenever Y_1 and Y_2 are substitutable and that (ii) \tilde{Y}_1 is an increasing function of Y_2 whenever Y_1 and Y_2 are complementary.

(b) $f^1(0, Y_2) = \bar{p}_1 < \infty$ and for $0 < Y_1 < \tilde{Y}_1$, $Y_2 \geq 0$, $f^1(\cdot)$ has continuous partial derivatives with the properties that

$$(c) \quad \partial p_1 / \partial Y_1 = f_1^1(\cdot) < 0 \text{ (downward sloping demand curve)}$$

$$(d) \quad \partial p_1 / \partial Y_2 = f_2^1(\cdot) < 0 \text{ (gross substitutes)}$$

$$(d') \quad \partial p_1 / \partial Y_2 = f_2^1(\cdot) > 0 \text{ (complements)}$$

$$(d'') \quad \partial p_1 / \partial Y_2 = f_2^1(\cdot) = 0 \text{ (unrelated products)}$$

$$(e) \quad \partial^2 p_1 / \partial Y_2^2 = f_{22}^1(\cdot) = 0$$

$$(f) \quad \partial^2 p_1 / \partial Y_1 \partial Y_2 = f_{12}^1(\cdot) = 0$$

The last two assumptions state that the substitution/complementarity effects are such that the relative shifts are by constant amounts and that the slope of the demand curve is unaffected by the presence of related goods within the firm respectively. These assumptions do not affect the generality of the results from the point of view of efficient choices of Y_1 and Y_2 within the firm as long as monotonicity in the shifts of the demand curves is assured. However, when the firm faces outside competition the presence of such related goods would affect the slopes of the demand functions and play a crucial role in deciding the firm's reaction to outside competition. Therefore these assumptions will be relaxed later.

In an analogous fashion the properties of the demand function for the second product Y_2 are given below:

Let $p_2 = f^2(Y_1, Y_2)$ represent the inverse demand function for Y_2 . The demand function is continuous and defined for all non-negative values of its arguments. Specifically,

(a) $f^2(\cdot) = 0$ for $Y_2 \geq \tilde{Y}_2$ given that $Y_1 > 0$. Again the value of \tilde{Y}_2 will depend on Y_1 in a fashion analogous to that postulated above while dealing with properties of the demand function for Y_1 .

(b) $f^2(Y_1, 0) = \bar{p}_2 < \infty$ and for $0 < Y_2 \leq \tilde{Y}_2$ and $Y_1 \geq 0$, $f^2(\cdot)$ has continuous partial derivatives with the properties that

$$(c) \quad \partial p_2 / \partial Y_2 = f_2^2(\cdot) < 0 \text{ (downward sloping demand curve)}$$

$$(d) \quad \partial p_2 / \partial Y_1 = f_1^2(\cdot) < 0 \text{ (gross substitutes)}$$

$$(d') \quad \partial p_2 / \partial Y_1 = f_1^2(\cdot) > 0 \text{ (complements)}$$

$$(d'') \quad \partial p_2 / \partial Y_1 = f_1^2(\cdot) = 0 \text{ (unrelated products)}$$

$$(e) \quad \partial^2 p_2 / \partial Y_1^2 = f_{11}^2(\cdot) = 0$$

$$(f) \quad \partial^2 p_2 / \partial Y_2 \partial Y_1 = f_{21}^2(\cdot) = 0$$

This completes the properties of the demand and cost curves that would be used in the model. In the next chapter, the economically efficient choices as well as the profit maximizing choices of specific product groups are analyzed.

EFFICIENCY OF HORIZONTAL INTEGRATION

The demand and cost aspects of the problem were outlined in the previous chapter. These would have an important bearing on the derivation of the efficient output quantities that ought to be produced by firms envisaging diversification activities. In this chapter a systematic analysis of the welfare maximizing and the profit maximizing choices of output quantities is undertaken and examined against the background of the concept of cost efficient levels of output quantities developed in Chapter Three. The analysis is done for three possible product groups. First is the case of a firm producing unrelated products. This is often called Conglomerate diversification. However, for purposes of comparison, this is considered as a special case of horizontal integration in this study. The other two possibilities examined are the cases of substitutes and complementary product lines.

4.1 The Welfare Maximizing Choices of Output Levels

Case I : Unrelated Products

Consider a firm producing a product Y_1 initially. Suppose it envisages the production of another product Y_2 which does not have a significant demand interrelationship with Y_1 . As argued earlier, it is assumed that there are economies of scope in the production of both these commodities together. Using the notations and properties of the demand

and cost structures developed in Chapter Three unrelated products would mean that assumptions (d'') on demand would apply. The total surplus function can be written as:

$$(4.1.1) \quad S(Y_1, Y_2) = \int_0^{Y_1} f^1(\theta) d\theta + \int_0^{Y_2} f^2(\bar{\theta}) d\bar{\theta} - C(Y_1, Y_2)$$

From the assumptions regarding the cost and demand structures, it is evident that the total surplus function is strictly concave, and hence a unique global maximum exists. Maximizing 4.1.1 yields the first and second order conditions:

$$(4.1.2) \quad S_1 = \frac{\partial S}{\partial Y_1} = f^1(Y_1) - C_1(Y_1, Y_2) = 0$$

$$(4.1.3) \quad S_2 = \frac{\partial S}{\partial Y_2} = f^2(Y_2) - C_2(Y_1, Y_2) = 0$$

$$(4.1.4) \quad S_{11} = \frac{\partial^2 S}{\partial Y_1^2} = f_1^1(Y_1) - C_{11}(Y_1, Y_2) < 0$$

$$(4.1.5) \quad S_{22} = \frac{\partial^2 S}{\partial Y_2^2} = f_2^2(Y_2) - C_{22}(Y_1, Y_2) < 0$$

$$(4.1.6) \quad S_{11}S_{22} - S_{12}^2 > 0 \quad \text{where} \quad S_{12} = \frac{\partial S_1}{\partial Y_2}$$

Totally differentiating 4.1.2 and 4.1.3 and rearranging the terms, it is clear that the locus of the (Y_1, Y_2) values satisfying 4.1.2 and 4.1.3 satisfy the equations:

$$(4.1.7) \quad \left(\frac{dY_1}{dY_2} \right)_{S_1=0} = \frac{C_{12}(Y_1, Y_2)}{\{f_1^1(Y_1) - C_{11}(Y_1, Y_2)\}}$$

$$(4.1.8) \quad \left(\frac{dY_1}{dY_2} \right)_{S_2=0} = \frac{\{f_2^2(\cdot) - C_{22}(\cdot)\}}{C_{21}(\cdot)}$$

The nature of the curves depicting the trajectory of the (Y_1, Y_2) values in the output plane depends crucially on the behaviour of the cost component $C_{12}(.)$. For, initially $C_{12}(.)$ is negative as there are economies of joint production. Therefore 4.1.7 and 4.1.8 would be positively sloped. At values of Y_1 and Y_2 for which $C_{12}(.) = C_{21}(.) = 0$, the slopes of the curves would be zero and ∞ respectively. For Y_1 and Y_2 such that $C_{12}(.) = C_{21}(.) > 0$, the slopes of both the curves would be negative. The intersection of the curves satisfying equations 4.1.7 and 4.1.8 would constitute the solution to the maximization problem. Algebraically, the intersection of S_1 and S_2 is possible at a point where $C_{12}(.) < 0$ or $C_{12}(.) = 0$ or $C_{12}(.) > 0$. These possibilities are depicted in Figure 4.1.0a,b,c. Each of these outcomes is dependent upon the market conditions for Y_1 and Y_2 respectively. An examination of equation 4.1.7 and 4.1.8 shows that the relative positions of the S_1 and S_2 curves are sensitive to the demand conditions. Two factors play a crucial role:

(a) The value of $f_1^1(.)$ ($f_2^2(.)$) and (b) the intercept of the S_1 (S_2) curve with the Y_1 (Y_2) axis. Given the cost curves, the values of these intercepts depend upon the 'position' of the demand curve for Y_1 (Y_2). A ceteris paribus change in the position of the demand curve for Y_1 (Y_2) changes the intercept of the S_1 (S_2) curve. Similarly, a ceteris paribus change in the slope of the demand function for Y_1 (Y_2) changes the slope of the S_1 (S_2) curve. For example, if the slope of the demand function given by $f_1^1(.) = -\alpha$ changes to -2α , then it is clear that the value of the intercept for S_1 is lower, and for some value of $C_{11}(.) = k$, $-\alpha - k > -2\alpha - k$ and hence the S_1 curve would be

flatter and strictly below the original S_1 curve. Figure 4.1.1a,b illustrate the above analysis for the S_1 curve. A similar interpretation would hold for the S_2 curve as well. It is clear that the three possibilities shown in Figure 4.1.0a,b,c are outcomes of different market conditions on Y_1 and Y_2 . For example, consider Figure 4.1.0a. Had the demand for Y_1 been higher and/or the slope of the demand function for Y_1 lower, then the possibility of a welfare maximising solution occurring at $C_{12}(.) = 0$ emerges. Figure 4.1.1c illustrates this.

To make matters concrete, consider the cost function[⊗]

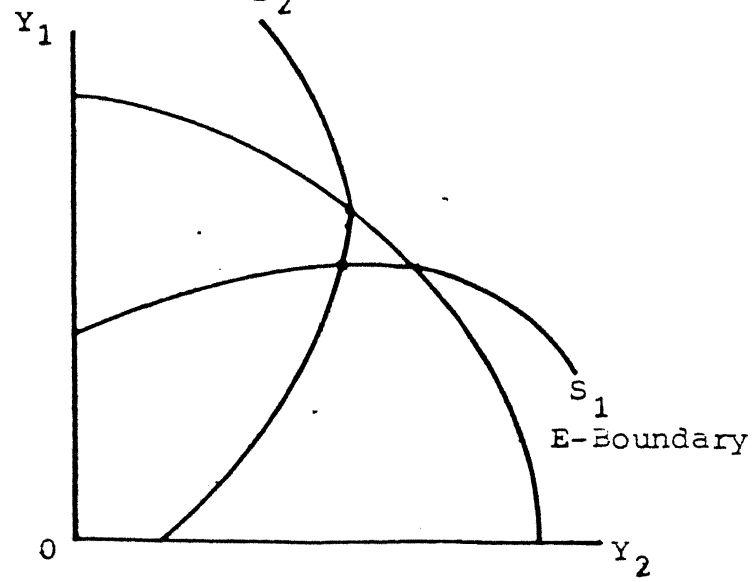
$$C = 4X - X^2 + \frac{1}{3} X^3 \quad \text{where} \quad X = \frac{1}{2} (Y_1^2 + Y_2^2)$$

$$\text{Then} \quad C_1 = (4 - 2X + X^2)Y_1$$

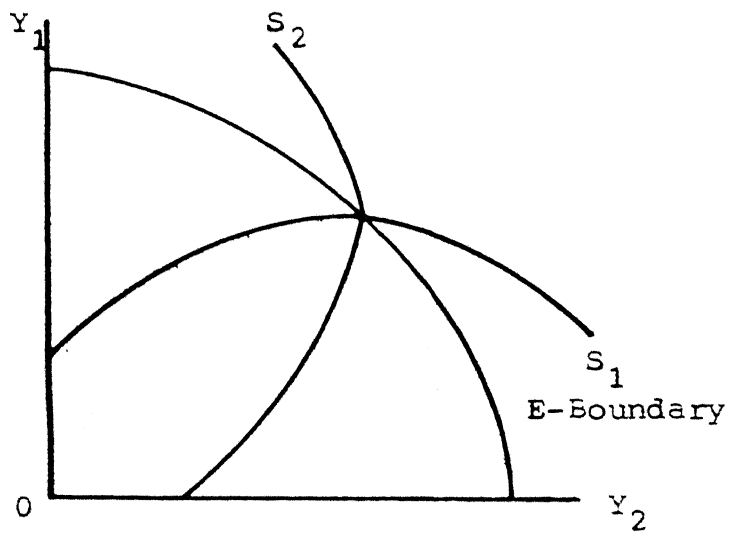
$$C_{12} = -2 Y_1 Y_2 (1-X) \quad \text{and} \quad C_{12}(.) \begin{matrix} < \\ > \end{matrix} 0 \quad \text{as} \quad X \begin{matrix} \leq \\ > \end{matrix} 1.$$

Let the demand curves for Y_1 and Y_2 be $p_1 = a - Y_1$ and $p_2 = b - Y_2$. Notice that if $a = b = 4$, the values of Y_1 and Y_2 satisfying equations 4.1.2 and 4.1.3 are $Y_1 = 1$, $Y_2 = 1$ and $X = 1$. Hence, $C_{12}(.) = 0$ is the welfare maximizing solution. However, $a = b < 4$, gives rise to welfare maximising solutions to be at $C_{12}(.) < 0$ and $C_{12}(.) > 0$ respectively. Similarly, if $a \neq b$, and $a < 4$, then there exists a value of b such that the solution Y_1, Y_2 would occur on the boundary. Let $a = 2.285$ approximately. Then, if $b = 4.840$ (approximately), $Y_1 = 0.5$, $Y_2 = 1.322$ and $(Y_1^2 + Y_2^2) \frac{1}{2} = 1$

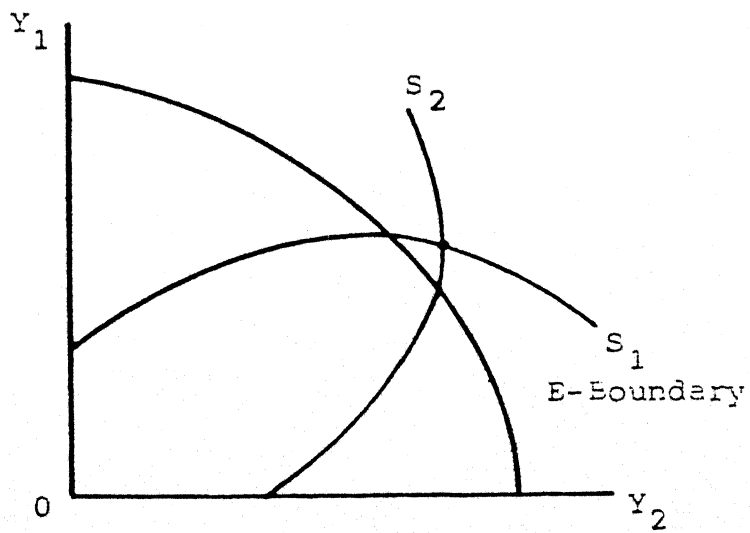
⊗ I am grateful to an anonymous referee for providing this example of a cost function while commenting on the thesis.



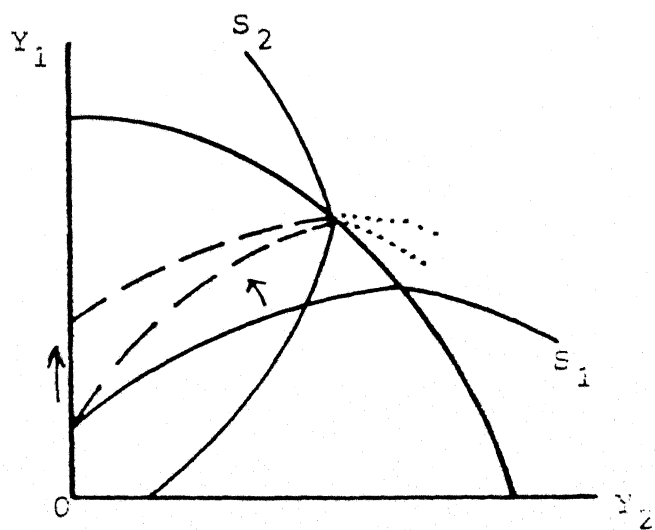
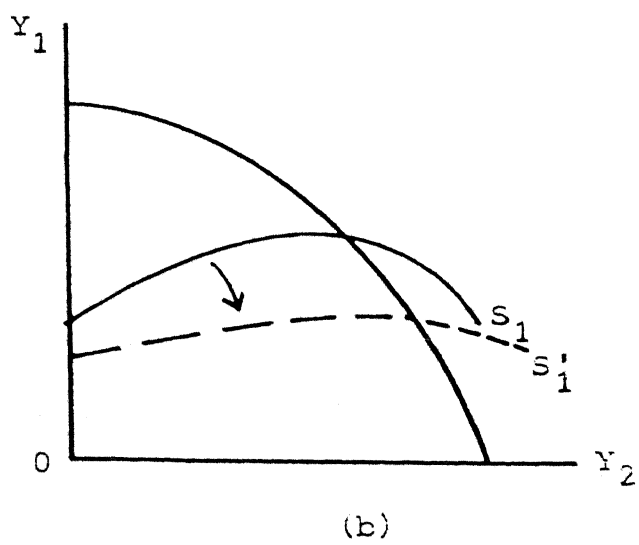
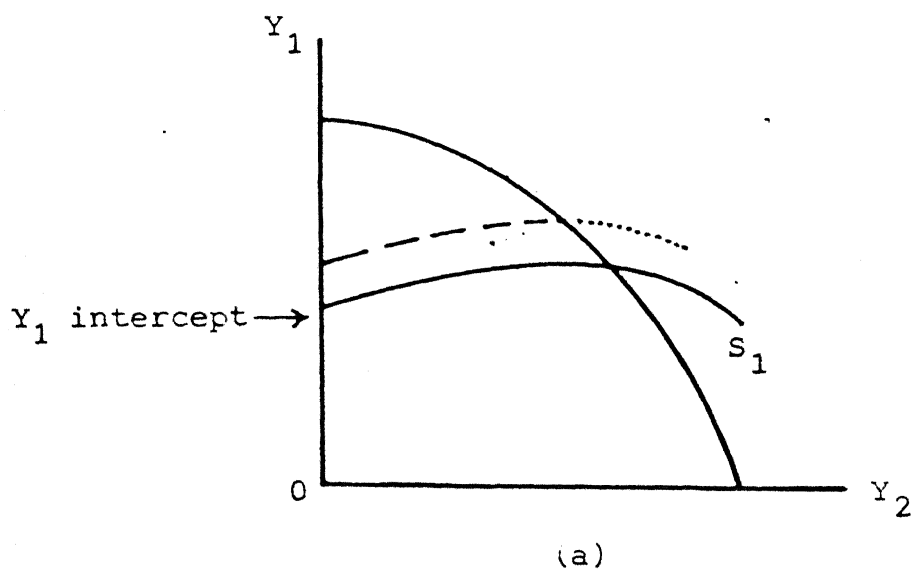
(a)



(b)



(c)



and $X = 1$ hence $C_{12}(.) = 0$ is satisfied. It is easily verified that the solution would occur in the $C_{12}(.) < 0$ region if $b < 4.840$, or would occur in the $C_{12}(.) > 0$ region if $b > 4.840$.

From this it can be concluded that there will be several combinations of 'a' and 'b' for which the welfare maximising choices of Y_1 and Y_2 satisfy $C_{12}(.) = 0$. However, it should be reiterated that this condition will not be satisfied by the welfare maximising values of Y_1 and Y_2 for all values of 'a' and 'b'. Hence, it can be concluded that there will be a class of demand curves $f^1(.)$ and $f^2(.)$ for which the welfare maximising choices of Y_1 and Y_2 will satisfy the cost-efficiency condition $C_{12}(.) = 0$. This can be stated as proposition 1.

PROPOSITION 1. There exists a specific class of demand curves for which the welfare maximizing choices of output levels by a firm undertaking the production of unrelated products would be cost-efficient as defined in Chapter 3, Section 3.3.

Figure 4.1.1d depicts such a solution with the optimal values represented by Y_1^e and Y_2^e respectively. An issue that deserves mention is that the proposition seems to suggest that the firm may in fact produce exactly two products. This is not true since the concepts of economies of joint production and economies of scope are much more general in nature. The more likely way of viewing multiproduct production is that of cost advantages obtained in the production of two or more products.

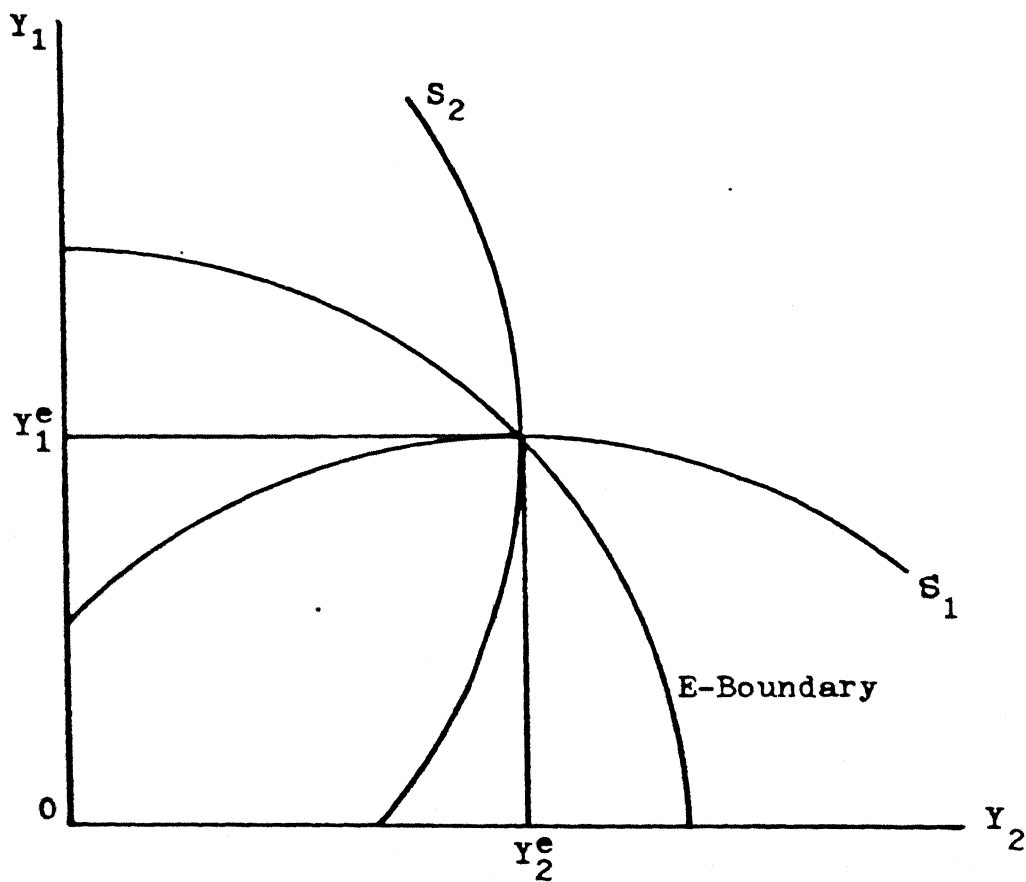


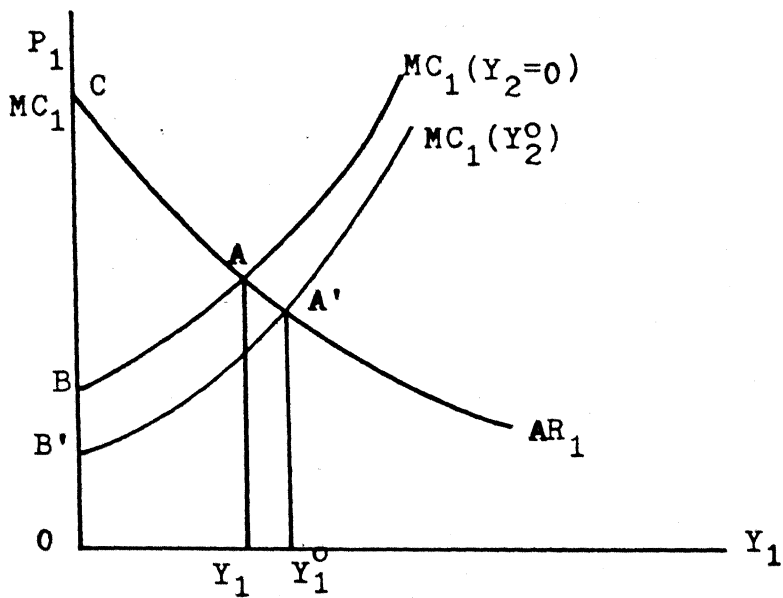
Figure 4.1.1 d.

An important corollary follows from proposition 1.

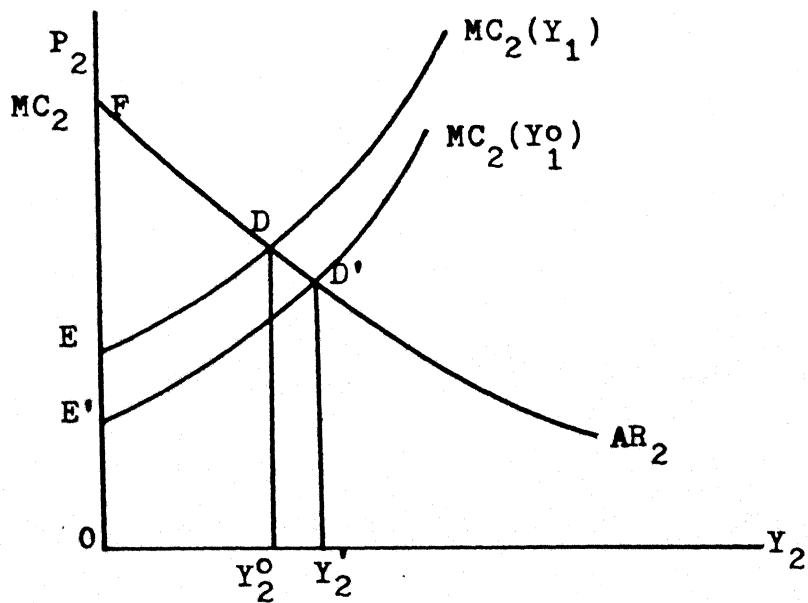
Corollary: Given the behaviour of costs as envisaged in Chapter Three, and that demand functions are given and fixed, the total surplus is a monotonic function of the output levels and varies directly with changes in output quantities.

The following economic reasoning helps demonstrate this corollary.

Let the firm be initially producing Y_1 only. AR_1 is the demand curve for Y_1 (which is the Average Revenue Curve) as shown in Figure 4.1.2(a) and $MC_1(Y_2 = 0)$ is the marginal cost curve for Y_1 when Y_2 was not produced. The welfare maximizing quantity is OY_1 . Given the existence of economies of joint production, the marginal cost curve for Y_2 becomes determinate when Y_1 is given. Let AR_2 be the demand curve for the second product. Y_2^0 is the equilibrium welfare maximizing level, as shown in Figure 4.1.2(b). Initially, the total surplus generated was equal to the area ABC. The production of Y_2^0 generates a surplus equal to the area DEF. However production of Y_2^0 would, on account of cost complementarity, reduce the $MC_1(Y_1)$ thereby generating the potential to increase production of Y_1 to Y_1^0 . The higher value of Y_1 corresponds to higher total surplus, the increase being equal to the area BAA'B'. If cost complementarity persists (that is, the E-boundary is not reached) then this increase in Y_1 to Y_1^0 will have a similar effect upon $MC_2(Y_1)$ and will therefore lead to a higher equilibrium value of $Y_2 = Y_2'$. This in turn contributes to the total surplus by the area EDD'E'



(a)



(b)

Figure 4.1.2.

(Figure 4.1.2(b)). If at these values cost complementarity is exhausted, the marginal costs do not change and total surplus remains the same. Any increase beyond this level would mean that the $MC_1(\cdot)$ and $MC_2(\cdot)$ would shift upwards towards the left and hence both the equilibrium values will go down. It is easy to see that the total surplus also goes down. Therefore, the total surplus increases or decreases as the equilibrium values of Y_1 and Y_2 increase or decrease. This property will be demonstrated to hold even when the demand functions shift, provided they shift monotonically. This is done at a later stage.

It is relevant to also observe that the total surplus generated by producing Y_1^e, Y_2^e jointly would be higher than the total surplus generated by producing Y_1^e, Y_2^e separately. Let S_j stand for the surplus under joint production and S_s under separate production

$$S_j = \int_0^{Y_1} f^1(\theta_1) d\theta_1 + \int_0^{Y_2} f^2(\theta_2) d\theta_2 - C(Y_1, Y_2)$$

$$S_s = \int_0^{Y_1} f^1(\theta_1) d\theta_1 + \int_0^{Y_2} f^2(\theta_2) d\theta_2 - [C(Y_1, 0) + C(0, Y_2)]$$

By lemma 1 Chapter Three, S_j is greater than S_s in the entire range within the E-boundary and $S_j(Y_1^e, Y_2^e)$ maximizes S_j , hence the result.

The case where the firm envisages selling products which show significant demand interrelationships (substitutability and complementarity effects) is now analyzed. In order to simplify the analysis and focus upon the effect of introducing such

interrelations upon welfare maximising choices of products, it is assumed that the class of demand functions are those which give rise to $C_{12}(.) = 0$ solutions as the welfare maximising choices of Y_1 and Y_2 in the independent demands case. However, Appendix A conjectures that the results proved in this chapter would hold in the other two cases where $C_{12}(.) < 0$ and $C_{12}(.) > 0$.

Case II : Y_1 and Y_2 Substitutable

In Chapter Three, it was seen that substitutable products may be a variety variant of a product type or may be varying in other product attributes so as to distinguish them from each other in the eyes of the consumer. Such demand interrelationships mean that the efficient choice of price/quantity of one of the products in the product line cannot be assessed in isolation.

Let $p_1 = f^1(.)$ and $p_2 = f^2(.)$ represent the demand functions for Y_1 and Y_2 respectively. The additional assumptions regarding the demand functions, (apart from assumptions a), b), c), e) and f) of Chapter Three) are: (a) The welfare maximising choices of Y_1 and Y_2 belong to the E-boundary (i.e., $C_{12} = 0$) when there is no substitutability. (b) Introducing demand substitutability amounts to assuming $\partial p_i / \partial Y_j = f_j^i(.) < 0$; $i \neq j$, $i = 1, 2$, $j = 1, 2$ (c) Integrability conditions are satisfied: The total surplus can be written as

$$(4.1.9) \quad S(Y_1, Y_2) = \int_L \left[\sum_{i=1}^2 \{ f^i(Y_1, Y_2) dY_i \} \right] - C(Y_1, Y_2).$$

Total consumers surplus is given by $\int_L \left[\sum_{i=1}^2 \{ f^i(Y_1, Y_2) dY_i \} \right]$, where \int_L is the line integral denoting the path of integration.

Observe that the surplus defined by 4.1.9 would yield an unambiguous measure of consumer surplus only if the integrability conditions are satisfied. This amounts to assuming:

$$(4.1.18) \quad \left(\frac{dY_1}{dY_2} \right)_{S_1=0} = - \frac{\{f_2^1(\cdot) - C_{12}(\cdot)\}}{\{f_1^1(\cdot) - C_{11}(\cdot)\}}$$

$$(4.1.19) \quad \left(\frac{dY_1}{dY_2} \right)_{S_2=0} = - \frac{\{f_2^2(\cdot) - C_{22}(\cdot)\}}{\{f_1^2(\cdot) - C_{21}(\cdot)\}}$$

The relationship between Y_1 and Y_2 is as follows:

Let $C_{12}(\cdot) < 0$ initially. Since $f_2^1(\cdot) < 0$ we must assume that $|C_{12}(\cdot)| > |f_2^1(\cdot)|$ for the firm to be horizontally integrated efficiently into the second product.

(i) When $|C_{12}(\cdot)| > |f_2^1(\cdot)| \Rightarrow \{f_2^1(\cdot) - C_{12}(\cdot)\} > 0$ and since the denominator of 4.1.18 is negative always, 4.1.18 is positive initially.

(ii) When $|C_{12}(\cdot)| = |f_2^1(\cdot)|$ for some higher values of Y_1 and Y_2 , 4.1.18 is zero.

(iii) When $|C_{12}(\cdot)| < |f_2^1(\cdot)|$, 4.1.18 is negative. The observed behaviour of 4.1.18 is depicted in Figure 4.1.3.

Notice that $|C_{12}(\cdot)| = |f_2^1(\cdot)|$ is satisfied for values of Y_2 to the left of A where cost complementarity is exhausted. The economic reasoning for the shape of the S_1 curve is straightforward.

Cost complementarity, that is, decreasing marginal cost of any one output for increases in the quantities of the other output has demand substitutability effects among the products in the product line as an opposing force. As long as the reductions in the marginal cost outweigh the loss in demand due to substitution relationships the equilibrium values of Y_1 and Y_2 would be higher than before. This is shown in Figure 4.1.4.

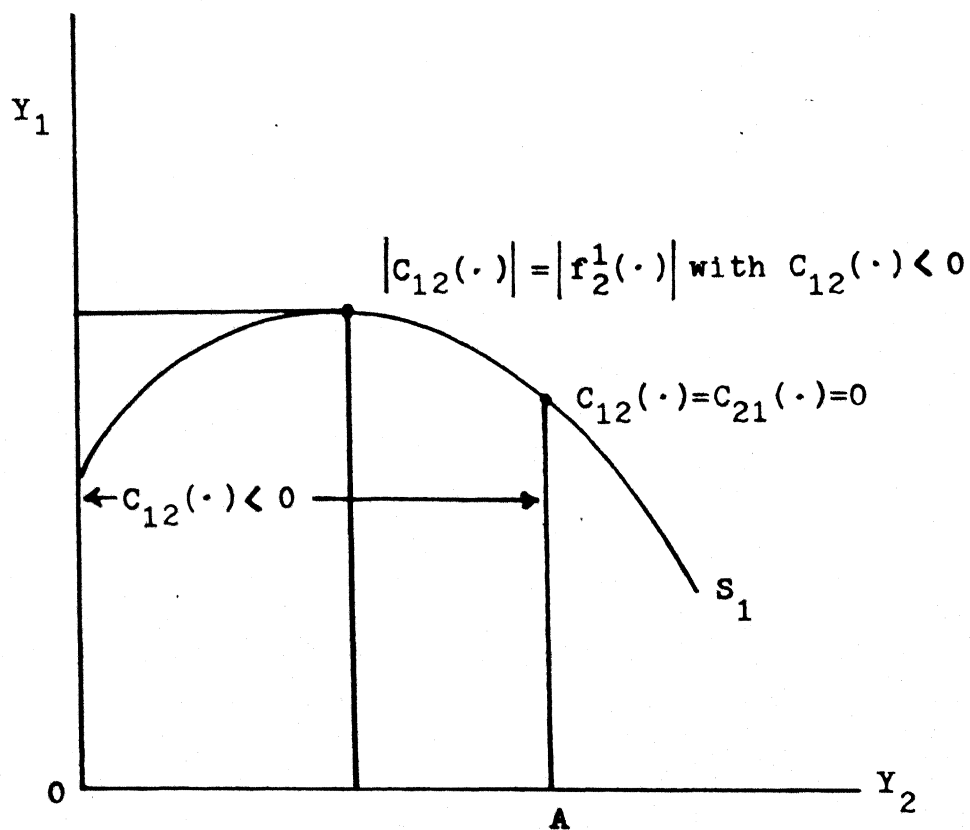


Figure 4.1.3

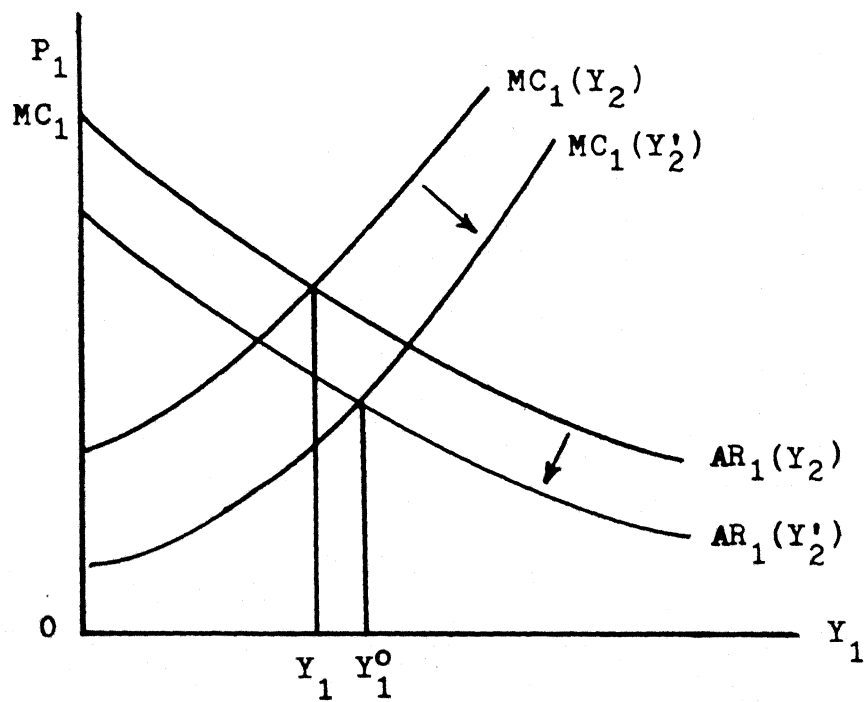
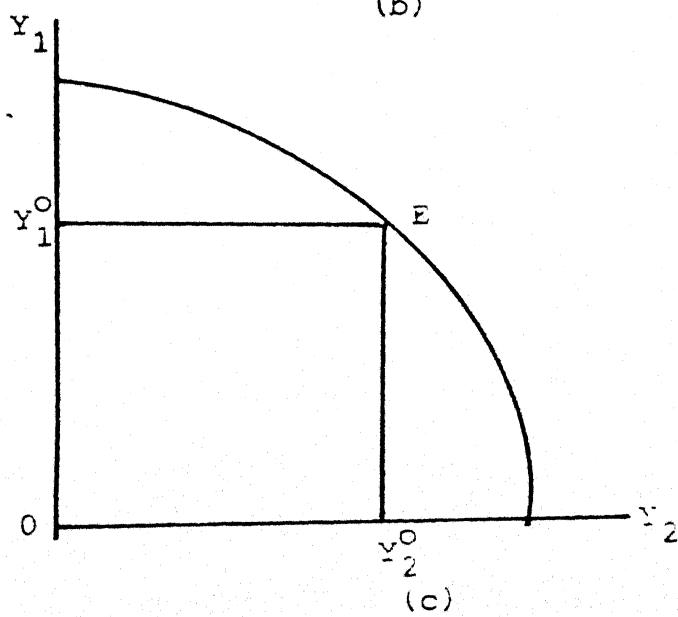
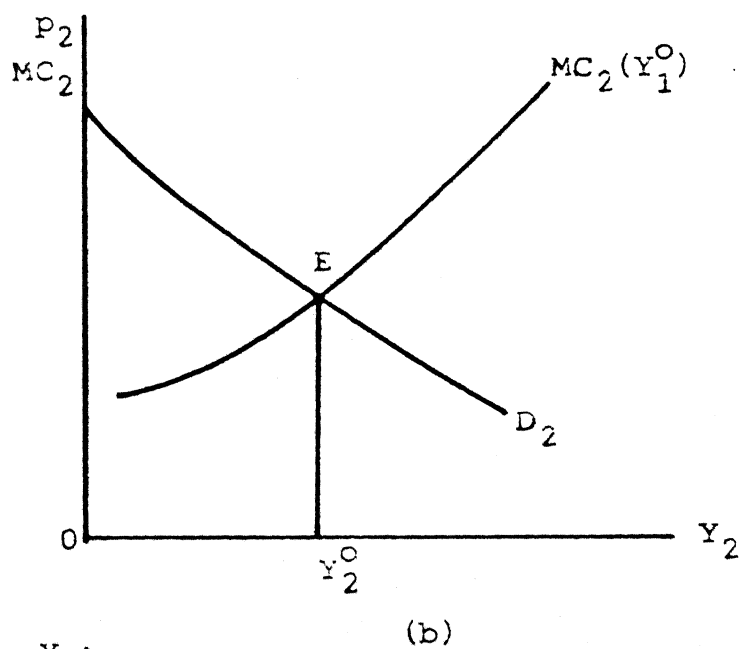
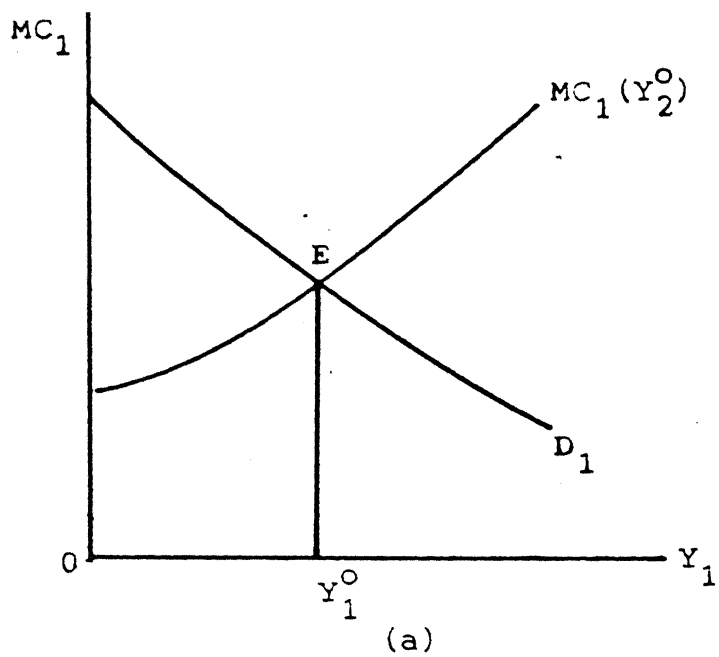


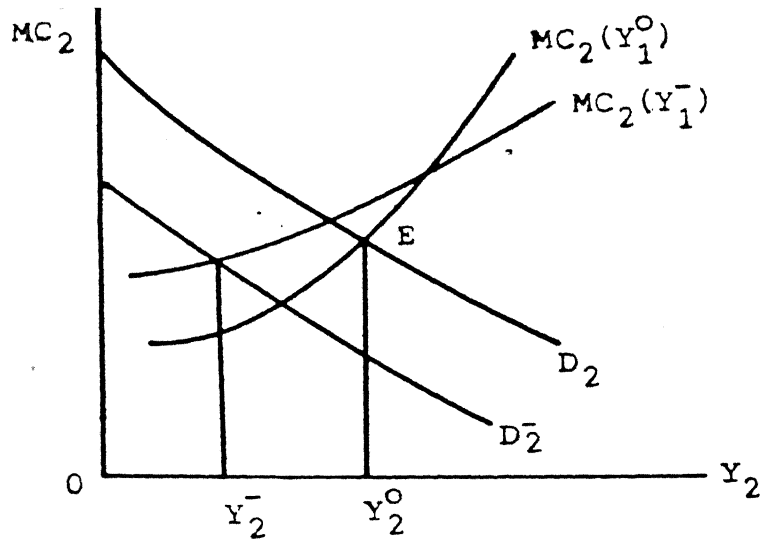
Figure 4.1.4.

Similarly, when $|C_{12}(\cdot)| = |f_2^1(\cdot)|$ the gains obtainable by reductions in costs are just offset by the losses due to the substitution effects in demand. When this substitution effect overtakes any cost advantages, an increase in quantity of one output will have the effect of decreasing the quantity of the other.

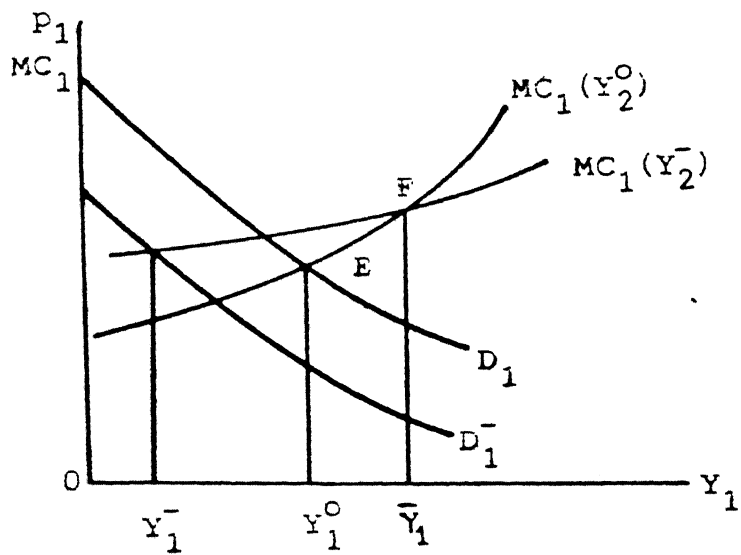
In an analogous fashion, the behaviour of 4.1.19 can be deduced. Using 4.1.18 and 4.1.19 the optimal values of Y_1 and Y_2 will be at the point of the intersection of S_1 and S_2 , respectively. The proof proceeds as follows:

Let D_1 and D_2 represent the demand curves which maximise surplus for values of Y_1^0 and Y_2^0 at which $C_{12}(\cdot) = 0$. The Figures 4.1.5a,b,c represent this. Point E in the Figures 4.1.5a and 4.1.5b corresponds to point E in Figure 4.1.5c. The introduction of substitutability between Y_1 and Y_2 entails a shift in the demand curves D_1 and D_2 which is downwards towards the left. A shift in the demand curve from D_2 to D_2^- results in a reduction of the equilibrium value of Y_2 . Consequently $MC_1(Y_2^0)$ would shift upwards, with $MC_1(Y_2^0) < MC_1(Y_2^-)$ evaluated at Y_1^0 . Using the properties of the marginal cost curves as envisaged in Chapter Three, that is Figure 3.3.1a, at some intermediate value of $Y_2 = \tilde{Y}_2 < Y_2^0$, and for $\bar{Y}_1 > Y_1^0$, $MC_1(Y_2^0) = MC_1(Y_2^-)$. This is shown in Figure 4.1.6b. Hence at (\bar{Y}_1, \tilde{Y}_2) , $C_{12}(\cdot) = 0$. Such a point corresponds to point F in Figure 4.1.6b. It is clear that such points are to the right of E. The equilibrium value of Y_1 in the substitutes case would be where $MC_1(Y_2^-)$ intersects D_1^- . It is seen that $Y_1^- < Y_1^0$. An analogous interpretation holds for the Y_2 market. Therefore both the equilibrium values would be lower than they

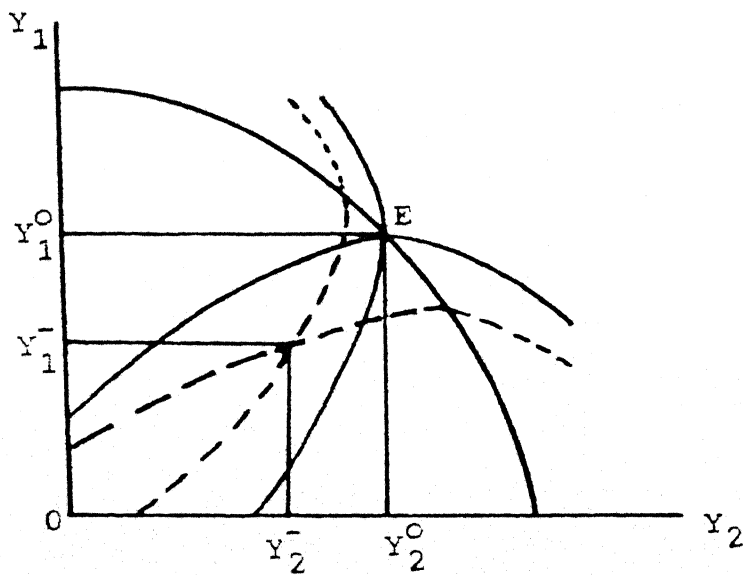




(a)



(b)



(c)

were when Y_1 and Y_2 were not substitutes. An example will demonstrate this. Let the demand curves be

$$P_1 = a_1 - b_1 Y_1 - d_1 Y_2$$

$$P_2 = a_2 - b_2 Y_2 - d_1 Y_1$$

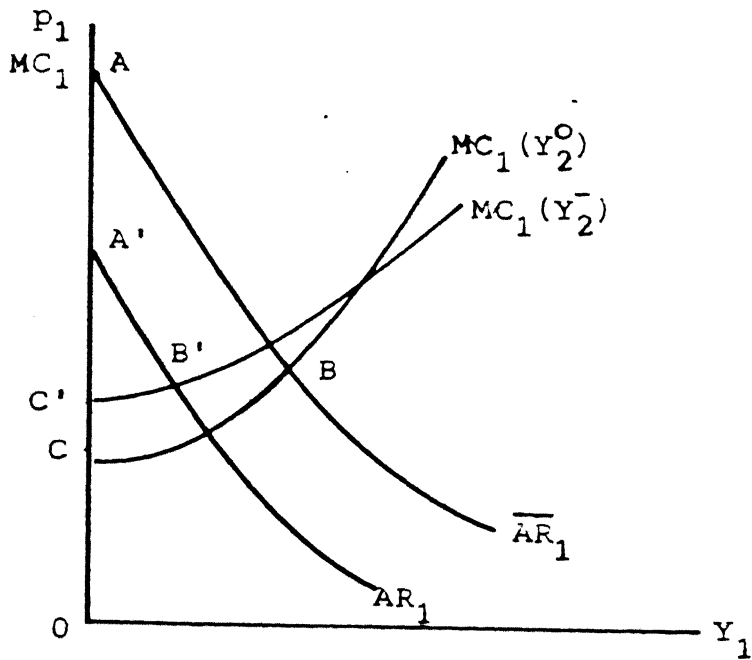
Let the cost function be $C = 4X - X^2 + \frac{1}{3}X^3$; ($X = \frac{1}{2}(Y_1^2 + Y_2^2)$).

Then $d_1 = 0$ represents the case where there are no substitutability effects. Further, let a_1, a_2 and b_1, b_2 take the values: $a_1 = a_2 = 4, b_1 = b_2 = 1$. It has been shown that if $d_1 = 0$ then maximising surplus gives quantities of Y_1 and Y_2 which are cost efficient i.e. $C_{12}(.) = 0$. Let Y_1 and Y_2 be substitutable instead. For example, let $d_1 = 1.76$. Then, the welfare maximizing choices of Y_1 and Y_2 , are: $Y_1 = Y_2 = 0.75$, and $X = 0.56$ and hence $C_{12}(.) < 0$.

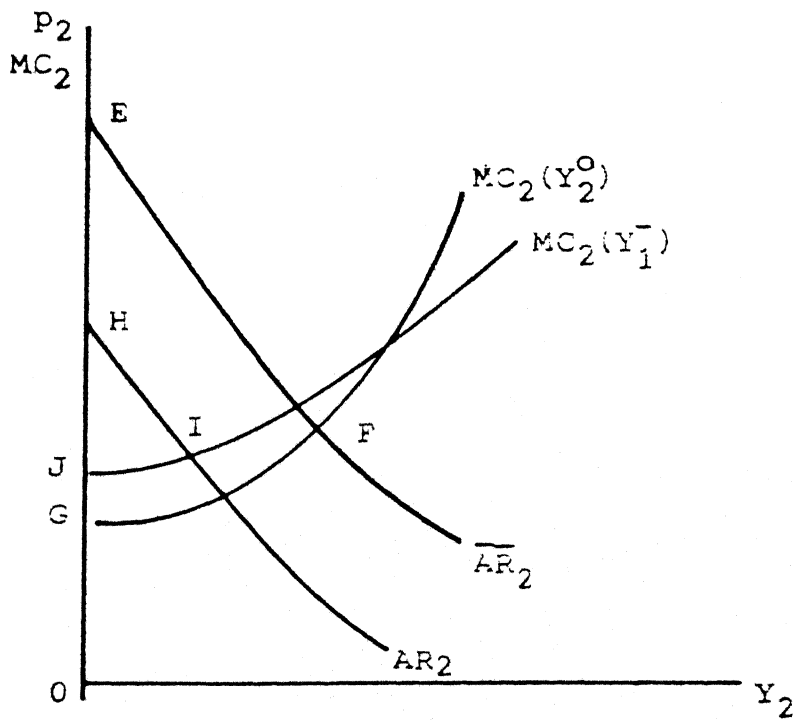
At a more general level, it can be argued that introducing substitutability in demand would lower the intercepts of both the S_1 and S_2 curves with the Y_1 and Y_2 axes respectively. Since the demand curves shift monotonically downwards the S_1 and S_2 curves would remain strictly below their original values. The relative positions of the curves is shown in Figure 4.1.6c.

It should be reiterated here that there exist demand functions, outside this class of functions, which will lead to welfare maximising values of Y_1 and Y_2 satisfying $C_{12}(.) = 0$. In such cases, the firm would in fact produce close substitutes in a cost-efficient manner. Some further observations are in order:

(A) The following welfare comparison can now be made between the product choices under propositions one and two. Consider Figures 4.1.7(a) and (b). The \overline{AR}_1 and \overline{AR}_2 curves are drawn under



(a)



(b)

Figure 4.1.7a,b

the assumption that Y_1 and Y_2 are independent in demand and that proposition one holds. The AR_1 and \bar{AR}_2 curves are drawn keeping in view the result of proposition two. It is clear that total surplus in the Y_1 market has gone down from the area ABC to A'B'C' and in the Y_2 market from the area EFG to HIJ. Notice that this loss in surplus is partly attributable to the increase in the marginal costs of both Y_1 and Y_2 .

E) Proposition 2 and its Corollary show that a product line consisting of gross substitutes precludes the firm from achieving the cost efficient output levels for its products. If we choose to ignore cost complementarities in production, then the cost efficient levels of Y_1 and Y_2 would be the same as those which maximize surplus. The notion of efficiency within a particular firm could not be evaluated through costs, since in following the marginal cost pricing rule, it would have to be assumed that the respective marginal costs represent the minimum possible in the neoclassical sense. Notice also that the existence of economies of joint production will always lead to an increase in welfare vis-a-vis production by independent units.

C) The above result also shows that the optimal values of Y_1 and Y_2 under demand substitutability do not fully exhaust economies of scope. This further provides an incentive for the firm to diversify into yet another product by using up the economies of scope provided that the new product does not adversely affect the demand for its existing produce lines. Spence (1976b) contends that a multiproduct firm will tend not to produce products that are close substitutes of each other. However, the reasons for this in his arguments are that gross substitutes will adversely affect the revenues accruing to each product in the product line. In the above model, the source of inefficiency originating from the demand side has a significant effect on the costs accruing to the firm, and hence, a recognition of this may cause the firm to behave in ways which are not optimal. To contend that

multiproduct firms will never produce products which are close substitutes of each other seems to be too strong a generalization. For, in the real world, firms do produce a variety of substitutable products. The following three possibilities seem to represent the more likely behaviour of firms:

(i) The optimal solutions that the firm would choose in selling substitutable products would presuppose a knowledge of the substitution effects by the firm. Given that the firms are usually in doubt even as to the demand curves for their products, there is a danger that demand may be ignored entirely in a multiproduct firm. This may be because the job of acquiring the necessary information may seem insurmountable. Therefore, the firm may well be choosing quantities of the products assuming that demand interrelationships are not significant, and so will logically use up the economies of scope that are available to it. If this happens, then the total achievable welfare would have to be redefined subject to the cost efficient choices of output levels. It is fairly obvious that the solution to the constrained welfare maximization problem would yield a total surplus less than that of the unconstrained welfare maximization problem.

(ii) On the other hand, if the firm does recognize the interdependencies of demand curves for its products in achieving cost efficient choices, it may well try to minimize these demand interrelationships. Resorting to market segmentation, or selling the two products to consumers 'groups' whose valuations of the products are sufficiently independent of one another, are two ways of achieving this.

(iii) The third possibility is further diversification into another product 'variety' of the existing products to cater to a section of the consumers who may have a lower valuation, and hence such a product would not have any 'near neighbour effects'.

Although the main propositions are proved only for the two products case, the general nature of the cost and demand structures enable us to reach very general conclusions regarding the possible behaviour of firms producing closely related products.

Case III : Complementary Products

Finally in deriving the welfare maximizing levels of a product line the case for complementary products is examined. It will be noticed that there are several types of complementary goods that need be specifically described even if it may not be possible to produce them together with much the same variable inputs. For example, the case of pens and ink. Although they are complementary products, there is no reason to believe that pens and ink have a common production base. At the other extreme is the case of joint products which will always be produced together. In between these two extremes may fall several varieties of complementary products which are more or less produced together because of certain common inputs that can readily be shared between them. An example would be the manufacture of recordplayers along with the manufacture of amplifiers and speakers. Similarly the manufacture of television sets along with VCRs' by the same

firm indicates the possibility of these two being produced together without the necessity of setting up a totally different plant and recruiting new expert personnel.¹

The present study of the case of complementary products addresses itself to those instances wherein there are economies of joint production in producing two products which are complementary to each other. In the sequel, it is argued that even if there are no economies of joint production and no economies of scope, production of complementary products by a single firm may still be preferred. The cost function will have the characteristics that an efficient-boundary will exist which delimits the region of economies of joint production. The total surplus will be written down as in 4.1.9 with the integrability conditions 4.1.10 assumed. We now invoke the assumptions that $f_j^i(.) > 0$ ($i \neq j, i, j = 1, 2$) on the demand functions.

Reconsider equations 4.1.18 and 4.1.19. Observe that for values of Y_1 and Y_2 where $C_{12}(.) \equiv C_{21}(.) \leq 0$, equations 4.1.18 and 4.1.19 will have a positive slope. When $C_{12}(.) = C_{21}(.) = 0$, then $|f_2^1(.)| = |C_{12}(.)| \Rightarrow$ equation 4.1.18 attains zero slope and is negative when $|f_2^1(.)| < |C_{12}(.)|$. Similarly we can deduce the behaviour of 4.1.19.

As in the case where Y_1 and Y_2 are substitutable it is assumed that the class of demand functions are such that $C_{12}(.) = 0$ is the welfare maximising solution when complementarity effects are absent. Consider the Figures 4.1.8a and 4.1.8b. D_1 and D_2 are the demand curves for Y_1 and Y_2 under independent demands.

¹Other examples may be that of fertilizers and insecticides; the furniture industry; or the garment industry.

The points E in both the figures correspond to a point on the E-boundary. If Y_1 and Y_2 were complementary then the demand for Y_1 would increase if Y_2 were also produced, which in turn increases demand for Y_2 . The relevant demand curves would then be D_1^+ and D_2^+ for Y_1 and Y_2 respectively. An increase in Y_2 (Y_1) leads to a higher value of $MC_1(Y_2^0)$ ($MC_2(Y_1^0)$) and therefore the point F where $MC_1(Y_2^0) = MC_1(Y_2^+)$ ($MC_2(Y_1^0) = MC_2(Y_1^+)$) would be such that $C_{12}(Y_1, Y_2) = 0$ and this would be to the left of E. Hence with the new demand and cost configuration, it is clear that Y_1^+ and Y_2^+ lie to the right of the boundary. Hence, at Y_1^+ and Y_2^+ , $C_{12}(Y_1^+, Y_2^+) > 0$. From this it can be concluded that:

PROPOSITION 3: Let $f^1(.)$ and $f^2(.)$ belong to the class of demand functions for which proposition one holds. If, however, Y_1 and Y_2 are complements in demand, and the analysis is confined to the above class of demand functions then the welfare maximising output levels will be above the cost efficient quantities.

The equilibrium solution given by the intersection of S_1 and S_2 is shown in Figure 4.1.8c.

The following observations are made:

A) It has been argued in the literature² that firms selling complementary products may indulge in 'loss leadership' which indicates practice of selling at prices below those levels at which they would have been sold had there been no complementary relationship. The above model clearly indicates that the quantities of Y_1 and Y_2 produced will be higher under demand complementarity conditions. However from this

²Bridge and Dodds (1975, pp. 235-236).

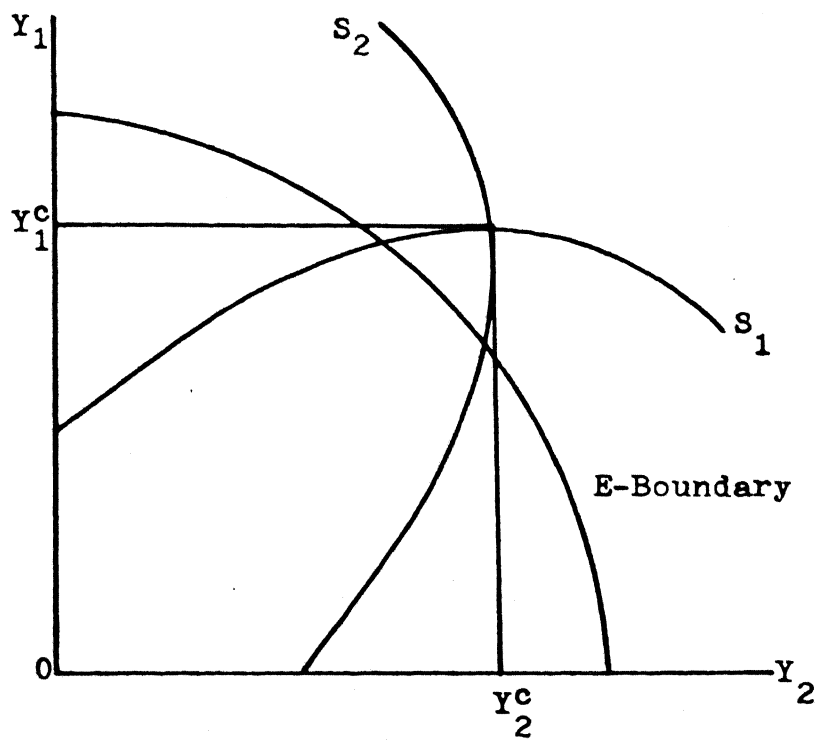
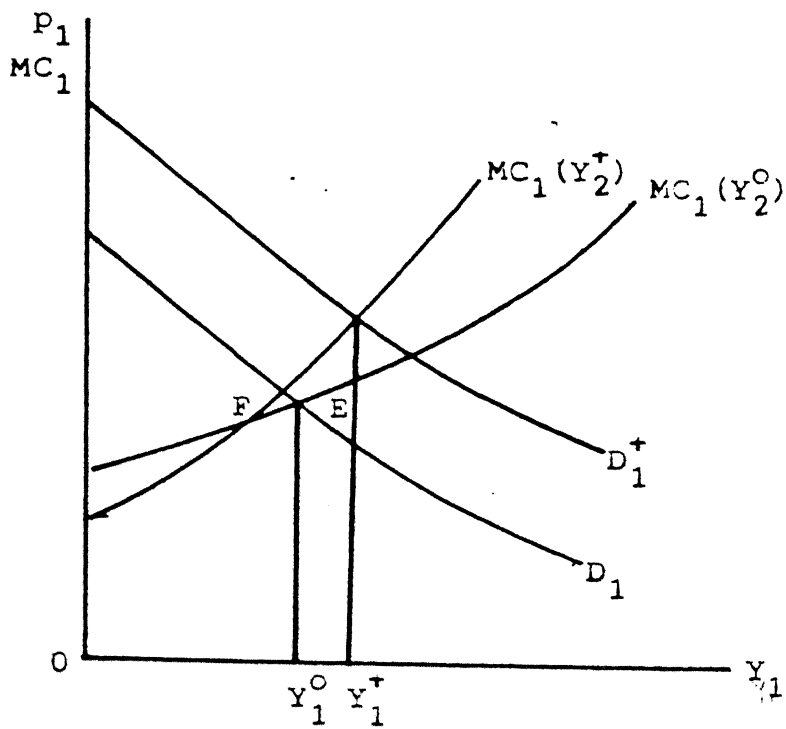
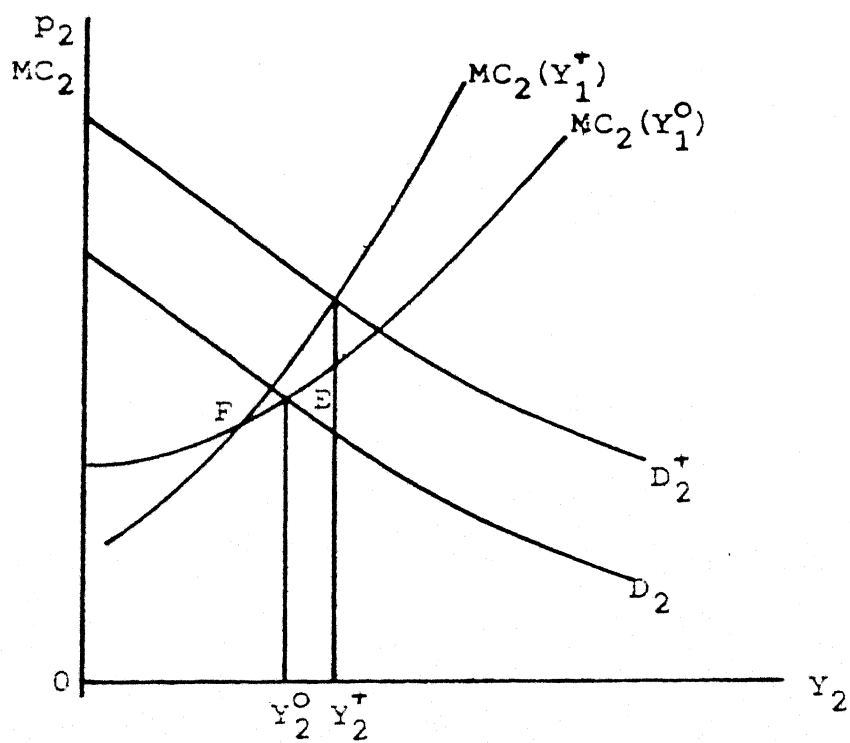


Figure 4.1.8.



(a)



(b)

Figure 4.1.8a,b

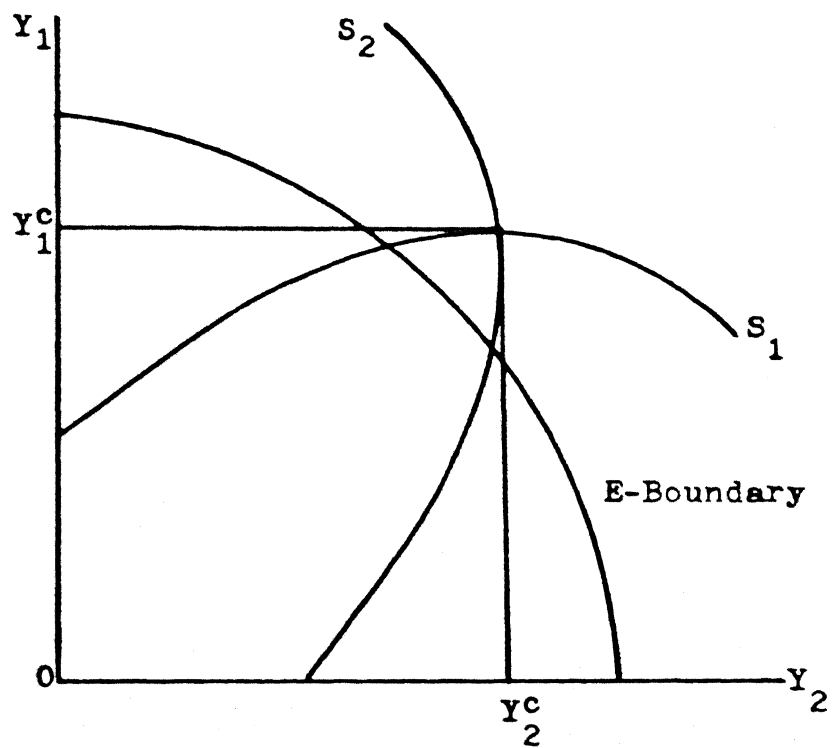


Figure 4.1.8c

it cannot be deduced that the prices will be lower since the new quantities and prices are not merely a result of a movement along a demand curve but due to shifts in the demand and cost structures.

B) Observe that the quantities of Y_1 and Y_2 produced by the firm are in the region where cost complementarity is exhausted. With the assumption of steeply increasing marginal costs of production, it may so happen that economies of scope do not extend into this range. Economies of scope, under such circumstances would be exhausted for values of Y_1 and Y_2 belonging to the E-boundary. However, since total surplus under joint production S_j is higher than under separate production S_s , the possibility that $S_s < S_j$ still exists even when economies of scope are exhausted. When economies of scope do exist beyond the E-boundary, the firm would produce in that region provided, (a) the demand for the products is large enough and (b) the ability of rival firms in the monopolistic environment to undercut is not significant.

There are other reasons beside these as to why complementary products would be horizontally integrated. Recall that the presence of transaction costs are a dimension of real life. Production of complementary items by separate firms entails costs of obtaining the necessary information about the demand curves of the complementary firm. This may, at best, be inadequately assessed. Horizontal integration would occur. Spence (1976b) suggests the same argument when he observed that monopolistic competition seems to be unsuited for complementary products.

This completes the analysis of the welfare maximising choices of a product line with and without demand interrelationships. Some of the features and results are now summarized:

(a) It is assumed that generating certain capacities and a commitment to certain capacity utilization (in both fixed as well non-product specific variable inputs) are dependent on certain expectations regarding the market conditions for the products that the firm wishes to sell. The incorporation of such 'public inputs leads to economies of joint production. The benchmark of efficiency from the firm's point of view is defined as production decisions which enable full exploitation of such cost advantages. However, if the ex-post demand for a product/s is lower than expected the firm may be precluded from exploiting the full range of available economies of scope. This is dependent upon the kind of changes in demand that are likely to occur.

(b) The welfare maximising choices of products which enable the firms to exhaust economies of joint production are dependent upon (i) the initial position of the demand curves, and (ii) the slopes of the demand curves and (iii) the substitution/complementarity effects on demand when the products in question are commercially related.

(c) An identification of the class of demand functions for which the welfare maximising solutions are cost-efficient was the objective of proposition one. Propositions two and three were then shown as cases where welfare maximum solutions do not correspond to cost-efficiency only due to substitution/complementarity effects.

(d) It is evident from proposition two, that total welfare would go down as compared to the benchmark quantities whereas complementary production may in fact increase welfare. But this increase is from the consumer side.

4.2 Profit Maximizing Choices of Output Levels:

In order to compare the profit maximising quantities of output levels vis a vis surplus maximising quantities the assumptions regarding the demand and cost-structures remain the same.

Case I : Unrelated Products

Let the demand curves faced by the firm belong to the same class as those in proposition one. The objective function to be maximised is now the total profit function. In carrying out the analysis of the profit maximising quantities of output levels, the following proposition holds:

PROPOSITION 4: Let $p_1 = f^1(Y_1)$ and $p_2 = f^2(Y_2)$ represent the demand functions for Y_1 and Y_2 respectively. Let $f^1(.)$ and $f^2(.)$ be so as to lead to cost-efficient solutions under welfare maximisation. Replacing the welfare maximisation objective by profit maximisation leads to quantities of both products to be lower than the surplus maximising quantities, thus resulting in cost-inefficiency.

Proof: The profit function is:

$$(4.2.1) \quad \pi(Y_1, Y_2) = Y_1 f^1(Y_1) + Y_2 f^2(Y_2) - C(Y_1, Y_2)$$

The first and second order conditions for a profit maximum are

$$(4.2.2) \quad \pi_1 = Y_1 f_1^1(Y_1) + f^1(Y_1) - C_1(Y_1, Y_2) = 0$$

$$(4.2.3) \quad \pi_2 = Y_2 f_2^2(Y_2) + f^2(Y_2) - C_2(Y_1, Y_2) = 0$$

$$(4.2.4) \quad \pi_{11} = Y_1 f_{11}^1 + 2f^1(Y_1) - C_{11}(Y_1, Y_2) < 0$$

$$(4.2.5) \quad \pi_{22} = Y_2 f_{22}^2 + 2f^2(Y_2) - C_{22}(Y_1, Y_2) < 0$$

$$(4.2.6) \quad \pi_{11}\pi_{22} - \pi_{12}^2 > 0$$

The relationship between Y_1 and Y_2 is obtained by totally differentiating 4.2.2 and 4.2.3 and rearranging the terms to yield the equations:

$$(4.2.7) \quad \left(\frac{dY_1}{dY_2} \right)_{\pi_1=0} = \frac{C_{12}(.)}{\{Y_1 f_{11}^1(.) + 2f_1^1(.) - C_{11}(.)\}}$$

$$(4.2.8) \quad \left(\frac{dY_1}{dY_2} \right)_{\pi_2=0} = \frac{\{Y_2 f_{22}^2(.) + 2f_2^2(.) - C_{22}(.)\}}{C_{21}(.)}$$

The behaviour of 4.2.7 and 4.2.8 depends upon the behaviour of $C_{12}(.)$. Initially, when $C_{12}(.) = C_{21}(.) < 0$, the slopes of the equations 4.2.2 and 4.2.3 in the Y_1 - Y_2 plane will be positive. Equation 4.2.2 would attain a zero slope when $C_{12}(.) = C_{21}(.) = 0$ and negative thereafter. Equation 4.2.3 would have an infinite slope at $C_{12}(.) = C_{21}(.)$ and negative thereafter. Consider the relative slopes of the equations 4.2.7 and 4.1.7; assuming that $f_{11}^1(.) = 0$ for simplicity, it is clear that the π_1 curve would (a) Have a smaller value of Y_1 as its intercept on the Y_1 axis than the corresponding S_1 curve and (b) Be flatter in slope than the corresponding S_1 curve. Similarly, the π_2 curve would have a smaller intercept on the Y_2 axis than the S_2 curve, but a steeper slope than the S_2 curve. However, since for all values of Y_1 (Y_2) the profit maximising quantities of Y_1 and Y_2 given by 4.2.1 and 4.2.2 are less than the corresponding surplus maximising quantities both the π_1 and π_2 curves would always be strictly below the S_1 and S_2 curves. It is evident that $Y_1^e > Y_1^\pi$ and $Y_2^e > Y_2^\pi$ and that $C_{12}(Y_1^\pi, Y_2^\pi) < 0$. The argument is shown in Figure 4.2.0.

As in the previous instances it must be noted that there would exist demand functions, outside the class being considered, which would allow the profit maximising choices to be cost-efficient.

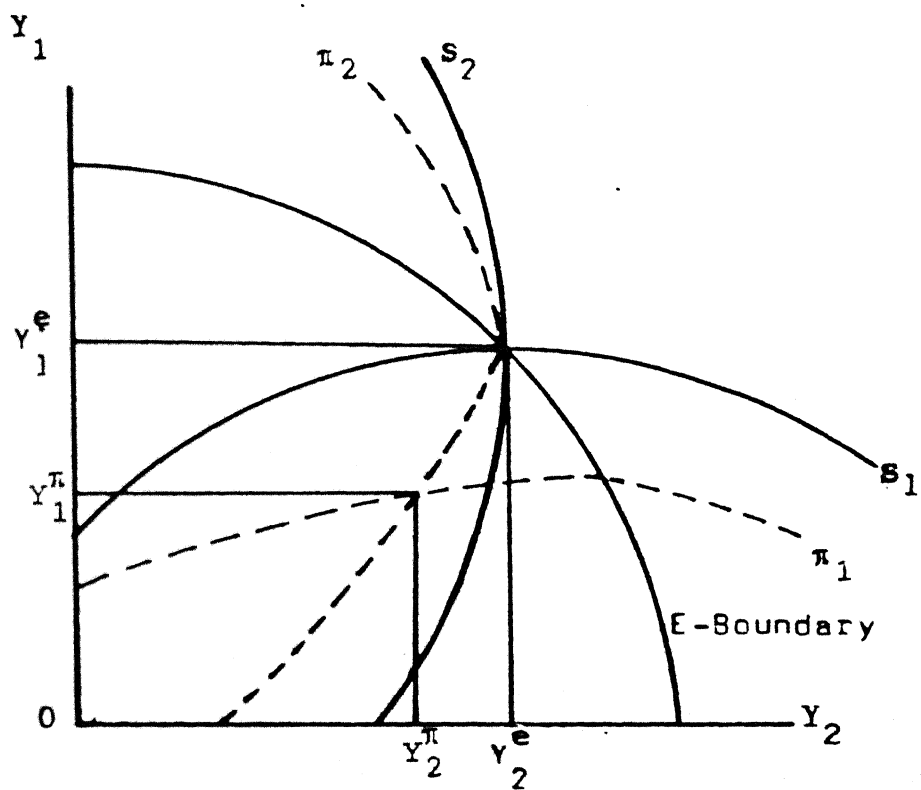


Figure 4.2.0.

Case II : Interdependent Products: Substitutes and Complements

It has been demonstrated that whenever the substitution relationship between one good and another good dominates, the changes in demand and costs conditions for the products will in turn, have complicated effects on the firm's total revenues and costs. From the point of view of efficiency within the firm, the following proposition holds in both the cases of substitutes and complements.

PROPOSITION 5: (a) Let the demand curves be such that the surplus maximising values of Y_1 and Y_2 under demand substitutability satisfy proposition two. Then, the corresponding profit maximising quantities would compound the inefficiencies caused by demand substitutability.

(b) Let the result of proposition 3 hold. Then the corresponding profit maximising choices would in fact be lower than before. The possibility of a cost-efficient solution cannot be ruled out.

Proof: The profit function can be generally written down as:

$$(4.2.9) \quad \pi(Y_1, Y_2) = Y_1 f^1(.) + Y_2 f^2(.) - C(.)$$

This function is strictly concave since if we assume for simplicity that the demand curves are linear, then,

$$\begin{aligned} H &= Y_1 f^1(.) + Y_2 f^2(.) \\ d^2 H &= Y_1 f_{11}^1(.) + 2f_1^1(.) + Y_2 f_{11}^2(.) dY_1^2 + 2 Y_1 f_{12}^1(.) + f_2^1(.) \\ &\quad + Y_2 f_{21}^2(.) + f_1^2(.) dY_1 dY_2 + Y_1 f_{22}^1(.) + Y_2 f_{22}^2(.) \\ &\quad + 2f_2^2(.) dY_2^2 < 0 \end{aligned}$$

For complementary products, the restriction is:

$$2f_1^1(\cdot)dY_1^2 + 2f_2^2(\cdot)dY_2 < [f_2^1(\cdot) + f_1^2(\cdot)]dY_1dY_2$$

The first and second order conditions for a maximum require that:

$$(4.2.10) \quad \pi_1 = \partial\pi/\partial Y_1 = Y_1 f_1^1(\cdot) + f^1(\cdot) + Y_2 f_1^2(\cdot) - C_1(\cdot) = 0$$

$$(4.2.11) \quad \pi_2 = \partial\pi/\partial Y_2 = Y_1 f_2^1(\cdot) + Y_2 f_2^2(\cdot) + f^2(\cdot) - C_2(\cdot) = 0$$

$$(4.2.12) \quad \pi_{11} = \partial^2\pi/\partial Y_1^2 = Y_1 f_{11}^1(\cdot) + 2f_1^1(\cdot) + Y_2 f_{11}^2(\cdot) - C_{11}(\cdot) < 0$$

$$(4.2.13) \quad \pi_{22} = \partial^2\pi/\partial Y_2^2 = Y_1 f_{22}^1(\cdot) + 2f_2^2(\cdot) + Y_2 f_{22}^2(\cdot) - C_{22}(\cdot) < 0$$

$$(4.2.14) \quad \pi_{12} = \pi_{21} = \partial^2\pi/\partial Y_1\partial Y_2 = Y_1 f_{12}^1(\cdot) + f_2^1(\cdot) + Y_2 f_{12}^2(\cdot) + f_1^2(\cdot) - C_{12}(\cdot) \\ = f_2^1(\cdot) + f_1^2(\cdot) - C_{12}(\cdot)$$

$$(4.2.15) \quad \pi_{11}\pi_{22} - \pi_{12}^2 > 0$$

Notice that equations 4.2.10 and 4.2.11 give the marginal revenue equals marginal cost rule. However, $Y_1 f_1^1(\cdot) + f^1(\cdot)$ is the marginal revenue obtained by selling one more unit of Y_1 . If Y_1 and Y_2 are substitutes, this would mean that the demand curve for Y_2 shifts down, and hence there is a loss in the total revenue obtainable from Y_2 . This is represented by $Y_2 f_1^2(\cdot)$ with $f_1^2(\cdot) < 0$. This entails an additional cost to be

borne by the firm. If Y_1 and Y_2 are complements, $f_1^2(\cdot) > 0$ and would in fact add to the revenue of the firm. At equilibrium, in the case of substitute products, marginal revenue must be equal to the marginal cost plus the loss entailed in the revenue obtainable in the Y_2 market. A similar interpretation can be given for equation 4.2.11.

The optimal values of the output levels is obtained by totally differentiating 4.2.10 and 4.2.11 and obtaining the trajectory of the paths of Y_1 and Y_2 satisfying the first- and second-order conditions.

$$(4.2.16) \quad \left(\frac{dY_1}{dY_2}\right) = - \frac{\{f_2^1(\cdot) + f_1^2(\cdot) - C_{12}(\cdot)\}}{\{Y_1 f_{11}^1(\cdot) + 2f_1^1(\cdot) + Y_2 f_{11}^2(\cdot) - C_{11}(\cdot)\}}$$

$$(4.2.17) \quad \left(\frac{dY_1}{dY_2}\right) = - \frac{\{Y_1 f_{22}^1(\cdot) + 2f_2^2(\cdot) + Y_2 f_{22}^2(\cdot) - C_{22}(\cdot)\}}{\{f_1^2(\cdot) + f_2^1(\cdot) - C_{21}(\cdot)\}}$$

The values of $f_2^1(\cdot)$ and $f_1^2(\cdot)$ are negative for substitutable products, and positive if they are complementary. Consider equation 4.2.16. Letting $f_{11}^2(\cdot) = 0$ and $f_{11}^1(\cdot) = 0$, the equation reduces to

$$\left(\frac{dY_1}{dY_2}\right) = - \frac{f_2^1(\cdot) + f_1^2(\cdot) - C_{12}(\cdot)}{2f_1^1(\cdot) - C_{11}(\cdot)}$$

Comparing with equation 4.1.18, it is seen that for any specific values of Y_1 and Y_2 in the region bounded by the E-boundary, Slope of equation 4.2.16 < Slope of equation 4.1.18

since $\{f_1^1(\cdot) - C_{11}(\cdot)\} > \{2f_1^1(\cdot) - C_{11}(\cdot)\}$

and $\{f_2^1(\cdot) + f_1^2(\cdot) - C_{12}(\cdot)\} < \{f_2^1(\cdot) - C_{12}(\cdot)\}$.

Hence the π_1 curve would have a smaller intercept with the Y_1 axis than the corresponding S_1 curve and be strictly below the S_1 curve. Consider equation 4.2.17 next. Comparing with equation 4.1.19, it is seen that the π_2 curve starts at a lower value of Y_2 and is steeper than the corresponding S_2 curve. However, it would be strictly below it (since marginal-revenue is always less than average revenue). The resulting values of Y_1 and Y_2 are shown in the Figure 4.2.1.

In the complements case also, the values of both Y_1 and Y_2 would come down for reasons analogous to the ones stated above. However, cost efficient solutions will be restored if and only if the corresponding marginal-revenue curves will so allow it.

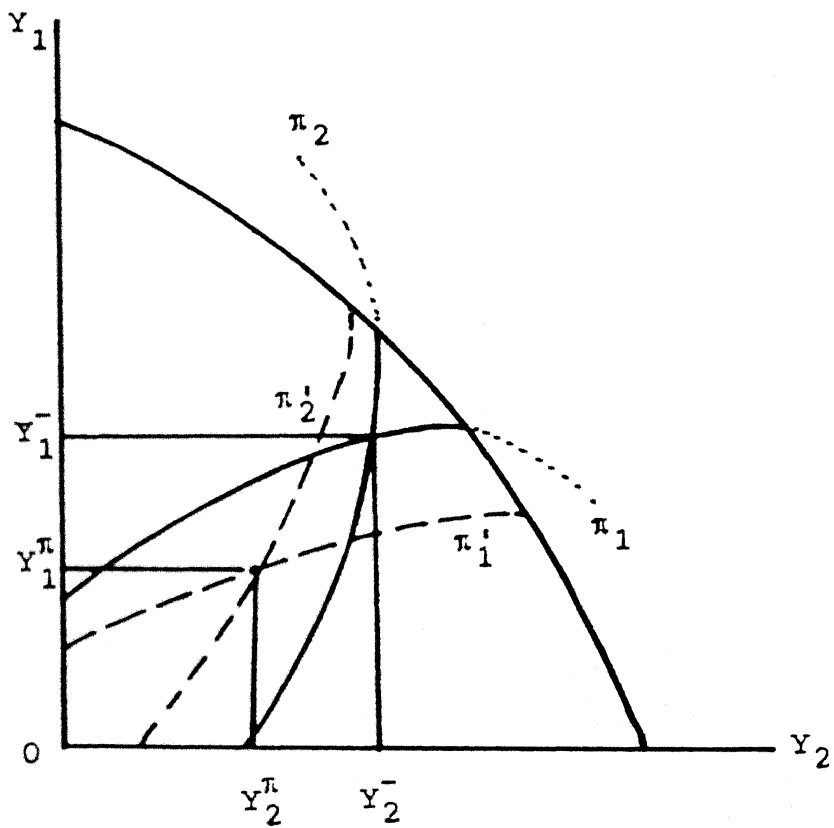


Figure 4.2.1

Looked at from another perspective, the above analysis would suggest that substitute products would be underproduced and complementary goods overproduced in a multiproduct firm. In this case the result in Spence (1976b) concerning complementary products is of particular interest. In his analysis, complementary products would be undersupplied in a monopolistically competitive environment; both from a point of view of profits and of total surplus. However, in the present multiproduct case, the analysis indicates that total surplus may actually go up. Product selection by the multiproduct firm, from the point of view of contributing to total surplus, is seen to include the production of substitutable products. However, in consonance with the literature, the elasticity of demand would still be a major consideration by which the firm may expand output levels of substitutable products that it chooses to produce. In this sense, any bias in selecting a product would be very much in accordance with the arguments in the literature. This aspect is pursued in the next chapter.

Lastly, since the above results purport to characterize the emergence of multiproduct firms through internal expansion, the selection of a product line would depend upon the type of economies of scope that are available to the firm and not simply the criteria of how much consumer surplus can be converted into profit. The behaviour of a multiproduct firm and its production decisions in a monopolistically competitive environment would perhaps be important enough to examine in the light of the literature on product choices under

monopolistic competition. The next chapter deals with the cases wherein a multiproduct firm faces competition for at least one of its products, and the resulting production level decisions of the firm are examined in the context of efficient choices as defined earlier.

CHAPTER 5

EFFICIENCY AND OUTSIDE COMPETITION

5.1 Introduction

Competition by an outside firm opens up many interesting possibilities regarding the effects upon the product-mix quantities chosen by a multiproduct firm. Given that demand interrelationships (whether substitutable or complementary in nature) would play a fundamental role in the non-fulfilment of cost efficient outcomes as shown in the previous chapter, the next logical step is to examine the efficiency of a firm when the demand curve for its products is affected by changes in the competitive level due to entry of other firms. It has already been seen in Chapter Four that introducing substitutability effects into the demand function, which would otherwise have allowed for cost efficient solutions to emerge, would result in a decrease in the output quantities of both products. Operating at levels where cost-complementarity is not exhausted would leave the marginal cost curves for either or both of the products at values higher than the minimum achievable levels. Economic intuition suggests that even

after the introduction of outside competition, inefficiency caused by interrelationships in demand of a product line when the products are substitutable may be perpetuated and that it (competition) may assist in an efficient solution occurring when the competition is for products that are complementary in nature. It is obvious that changes in the competitive level has two significant effects upon the demand curves for the existing products. (a) The elasticity of demand at any given point would change whenever the competitive level changes. It is clear from the literature that the demand curve for the monopolistic producer would be more elastic as the number of firms increase. (b) It has also been argued in Chapter Three, Section 3.4, that changes in the competitive level would have the effect of making the demand curve steeper whenever consumers with a lower valuation of the firm's product switch to the competing firm.

In the analysis that is to follow, both the above possibilities are incorporated. It will be shown that the relative steepness of the demand curve plays an important role in deciding both the welfare maximizing as well as the profit maximizing choices of output levels of the firm facing competition due to entry. However, the elasticity of demand argument plays an important role in the profit maximizing case. Since the analysis is concerned only with the effect of a change in the competitive level on the efficient choice of output levels, the class of demand functions assumed are those which allow cost-efficient solution to hold in both welfare maximising as well as profit maximising quantities of Y_1 and Y_2 .

Changing competitive level is assumed to be given exogenously. The case where the multiproduct firm faces competition from other firms and the effects upon its own product line, wherein the products are substitutes has been analyzed by Katz (1984) and Ireland (1983). Although Katz (1984) did not assume the existence of economies of scope in the production process, it is clear that an explicit modelling of the consumers preferences is necessary to obtain insights into the precise nature of the demand interrelationships so generated. This is considered beyond the scope of the present study. Instead, the present analysis concentrates on the effect of competition on (a) unrelated products and (b) complementary products produced by the multiproduct firm. This is done in Section 5.2. The effects on profit maximizing choices of output quantities, when the products are unrelated is examined in Section 5.3. This enables us to focus upon the issues of product selection that are argued in the literature. These are the subject matter of Section 5.4.

5.2 Competitive Effects on the Welfare Maximizing Levels of Products in a Product Line

Case I : Unrelated Products

Let the representative firm produce two products Y_1 and Y_2 which exhibit cost interrelationships. It is assumed that Y_1 and Y_2 are unrelated in demand. Let the quantity of the competitive product be X , which is a gross substitute of Y_1 . Specifically, let $p_1 = f(Y_1, X)$ represent the demand function for product Y_1 faced by the firm. $p_2 = g(Y_2)$ is the demand

function for the firm's second product. It is assumed that the quantity X of the competitive product is exogenously given. The following properties of demand are assumed.

$$(a) \quad \partial p_1 / \partial Y_1 = f_1(Y_1, X) < 0 \quad (\text{downward sloping demand curve})$$

$$(b) \quad \partial p_1 / \partial X = f_2(Y_1, X) < 0 \quad (\text{gross substitutes})$$

$$(c) \quad \partial^2 p_1 / \partial Y_1 \partial X = f_{12}(Y_1, X) \leq 0$$

i.e., the demand curves may get flatter due to the availability of a larger number of substitutes in the traditional Chamberlinian sense, or may in fact become steeper as argued earlier.

$$(d) \quad \partial p_2 / \partial Y_2 = g'(Y_2) < 0 \quad (\text{downward sloping demand curve})$$

As before, let $C(Y_1, Y_2)$ represent the total cost function with the properties assumed in Chapter Three, Section 3.3. The surplus function of the representative firm is,

$$(5.2.1) \quad S(Y_1, Y_2, X) = \int_0^{Y_1} f(\theta_1, X) d\theta_1 + \int_0^{Y_2} g(\theta_2) d\theta_2 - C(Y_1, Y_2)$$

where, X is the exogenously given quantity of the competitive product. Variations in X are also assumed to take place exogenously. Assuming the demand conditions under which proposition one holds, the following proposition can be proved:

PROPOSITION 6: A multiproduct firm facing an increase in the competitive level for one of its products will cause a deviation from cost efficient choices whenever competition

makes the demand curve for the product flatter or whenever competition tends to take away consumers with lower valuations of the product.

Proof: The proposition can be demonstrated as follows.

Maximizing 5.2.1, the first order conditions yield,

$$(5.2.2) \quad S_1 = \partial S / \partial Y_1 = f(Y_1, X) - C_1(Y_1, Y_2) = 0$$

$$(5.2.3) \quad S_2 = \partial S / \partial Y_2 = g(Y_2) - C_2(Y_1, Y_2) = 0$$

The second order conditions for a maximum are assumed to hold throughout. According to Proposition 1, Chapter Four, Section 4.1, the optimal values of Y_1 and Y_2 would lie on the E-boundary. That is, these values are so as to exhaust any cost complementarities in the production of Y_1 and Y_2 . Proposition 6 can be demonstrated in two parts, (i) when the demand curve for the product becomes flatter with competition and (ii) when it becomes steeper with competition.

Observe that the initial choice of Y_1 for $Y_2 = 0$ will always be less when X is positive, since the limit $Y_2 = 0$,

$$(5.2.4) \quad \left(\frac{dY_1}{dX} \right)_{Y_2=0} = - \frac{f_2(Y_1, X)}{f_1(Y_1, X) - C_{11}(Y_1, 0)} < 0$$

since both numerator and denominator are always negative.

The implication of equation 5.2.4 is that the intercept of the S_1 curve on the Y_1 axis shown in Figure 5.2.0 would be at a lower value of Y_1 when X changes exogenously. The relative change in the value of the intercept depends upon the substitution effect, $f_2(Y_1, X)$; the greater is the magnitude of $f_2(\cdot)$,

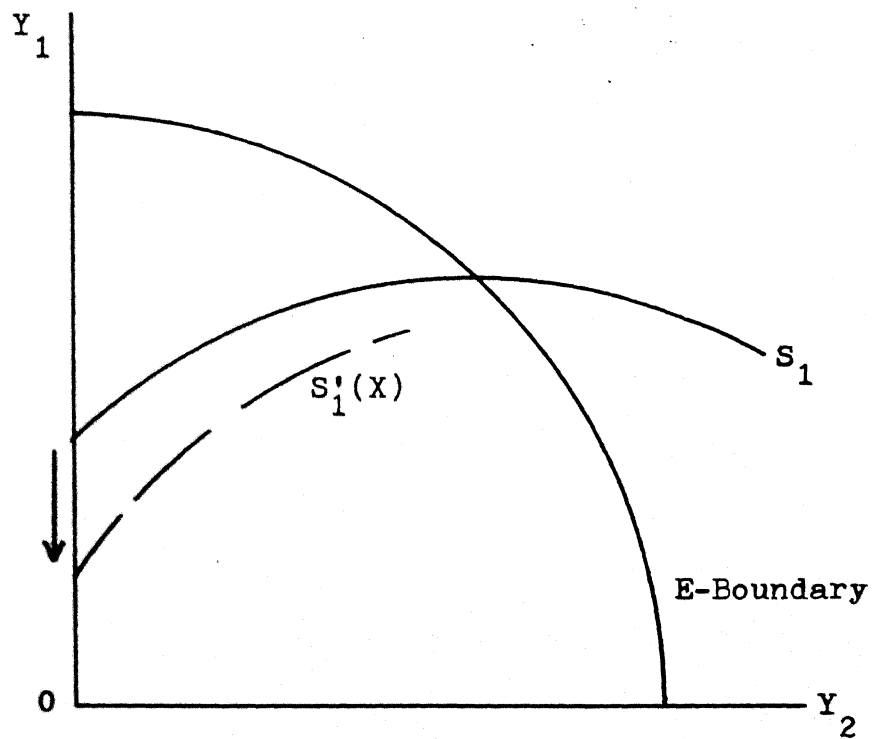


Figure 5.2.0

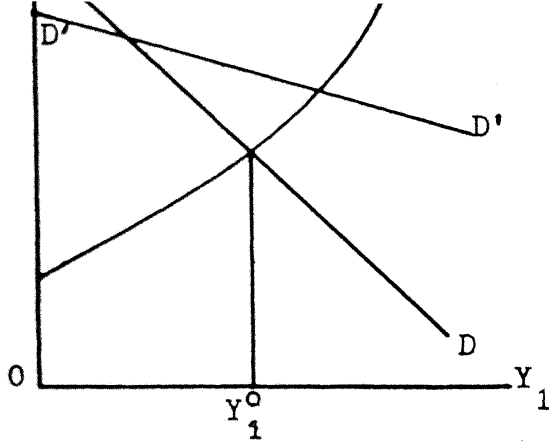
the lower would be the intercept value of Y_1 . Next, the change in the slope of the S_1 curve when X changes exogenously is given by the differential,

$$(5.2.5) \quad \frac{\partial}{\partial X} \left[\frac{C_{12}(Y_1, Y_2)}{[f_1(Y_1, X) - C_{11}(Y_1, Y_2)]} \right]$$

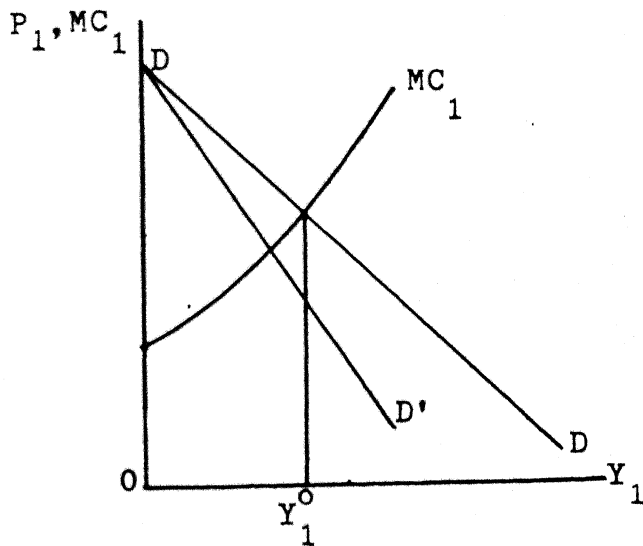
let $[f_1(\cdot) - C_{11}(\cdot)] = A$. $A = A(X)$, since this is a function of X . Evaluating 5.2.5 yields,

$$(5.2.6) \quad - \frac{C_{12}(\cdot) f_{12}(\cdot)}{(A(X))^2}$$

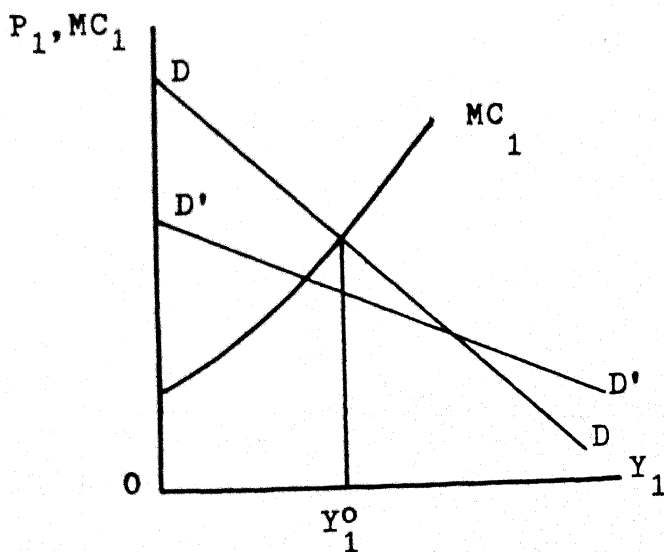
To demonstrate the first part of the proposition, let $f_{12}(\cdot) > 0$ always and $f_2(\cdot) < 0$. When $C_{12}(\cdot) < 0$ initially, the change in the slope of S_1 given by equation 5.2.6 is positive. It becomes negative for all values of $C_{12}(\cdot) > 0$. Since the S_2 curve is not a function of X , it remains unchanged. From equations 5.2.4 and 5.2.6, it can be deduced that the S_1 curve would have a lower value of Y_1 as the intercept and would be steeper than it was previously, in the range bounded by the E-boundary. The restrictions on the system are $f_{12}(\cdot) > 0$ and $f_2(\cdot) < 0$ in the relevant neighbourhood of Y_1 , and therefore, the shifts in the demand function must be so as to fulfil both these assumptions simultaneously. Consider the Figures 5.2.1(a,b,c). It is clear that a shift in the demand curve of the type envisaged in Figure 5.2.1(c) would be the only relevant way of interpreting the assumptions. The relative shift in the slope of the S_1 curve can be shown to exhibit any one of the following possibilities, shown in Figures 5.2.2(a,b,c) if one goes by equations 5.2.4 and 5.2.6.



(a). $f_{12}(\cdot) > 0$ but $f_2(\cdot) < 0$



(b). $f_2(\cdot) < 0$ but $f_{12}(\cdot) > 0$



(c). $f_{12}(\cdot) > 0$ & $f_2(\cdot) < 0$

Figure 5.2.1.

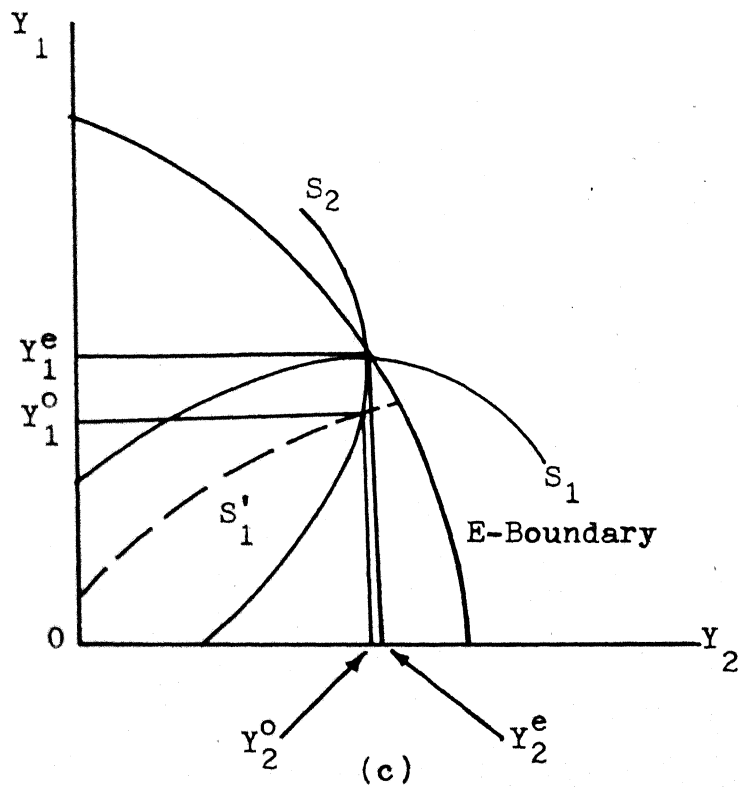


Figure 5.2.2

However, it is easy to see that the economic reasoning behind these possibilities depicted in Figure 5.2.2(a,b) leads to the conclusion that they are valid only if the demand curves shift in such a way so as to violate the assumption $f_2(\cdot) < 0$. This is shown in Figure 5.2.3(a,b). Therefore, the only solution would be that Y_1 and Y_2 reduces, as shown in Figure 5.2.2(c).

Q.E.D.

The economic reasoning for this outcome is as follows.

Referring to Figure 5.2.2(c), the possibility shown therein refers to inefficiency caused within the firm whereby outside competition makes the firm reduce the production of Y_1 . Since significant cost interrelationships are postulated, the effect of a reduction in Y_1 (which will be called the 'direct effect' due to changes in the competitive level also results in a contraction in Y_2 , although to a smaller extent. This 'indirect effect' occurs even though Y_2 is not related to the commodity X. The more important part of the analysis is the mechanism by which this result may emerge. There are a multitude of factors that play an important role under these circumstances. For, observe that the significant effect of changes in the competitive level has a bearing on the demand curve for the product in question. Therefore, a change in the equilibrium configuration is expected. At the same time, changes in the existing values of Y_1 and Y_2 would also mean that the marginal cost curves would change. Therefore the final outcome depends upon the relative shifts in both the demand as well as the marginal cost curves. Let DD represent

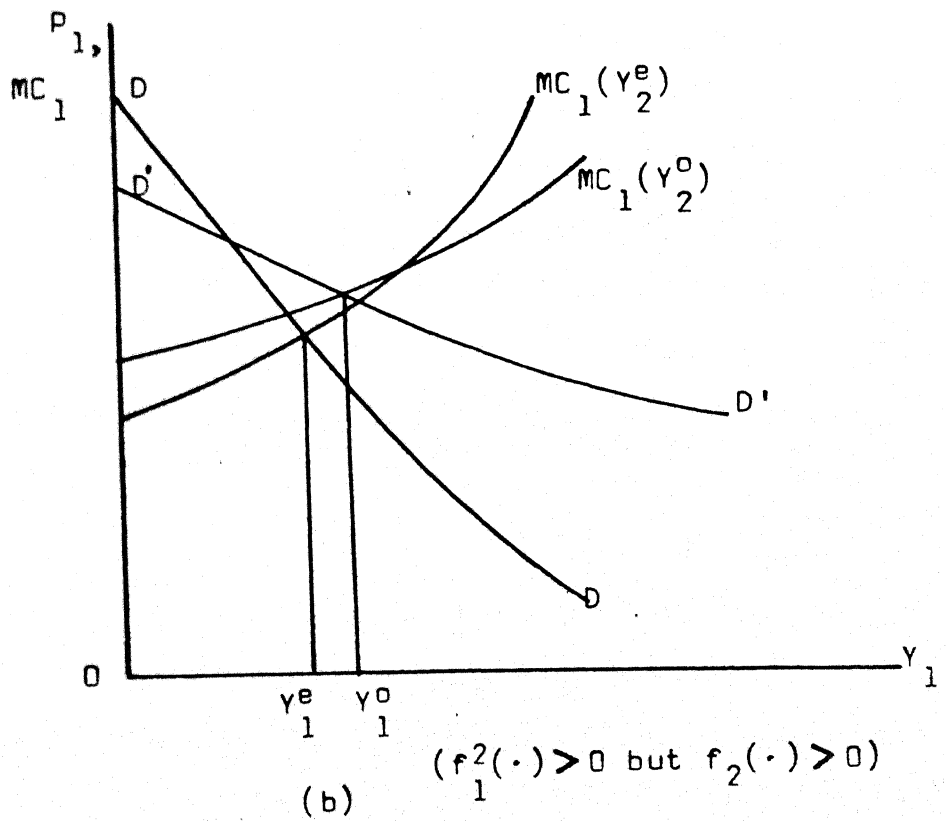
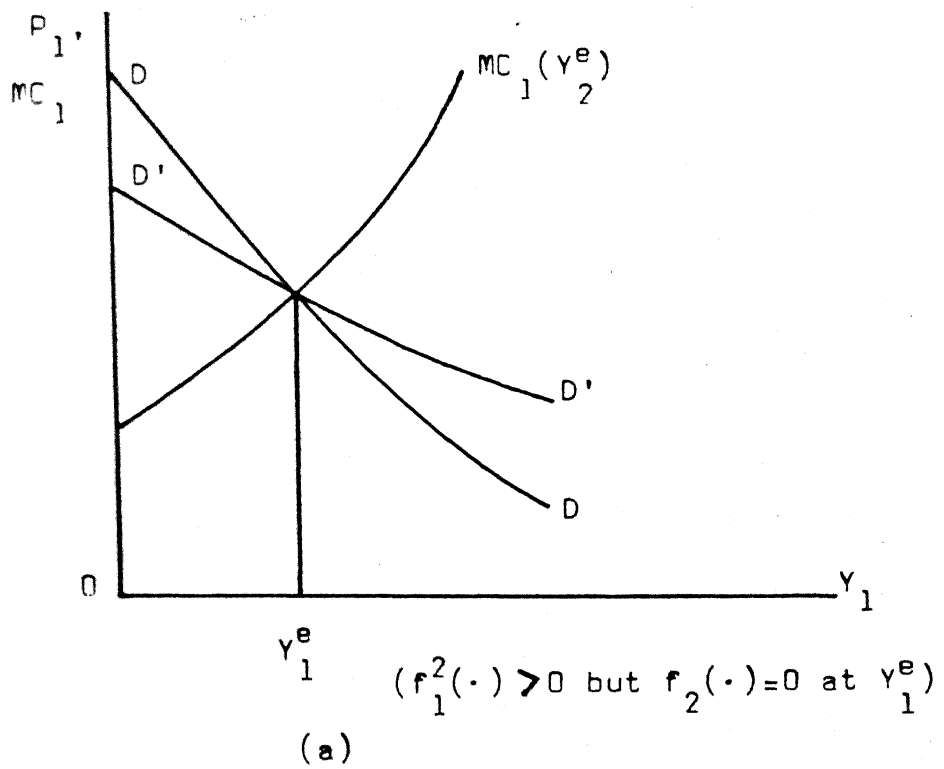
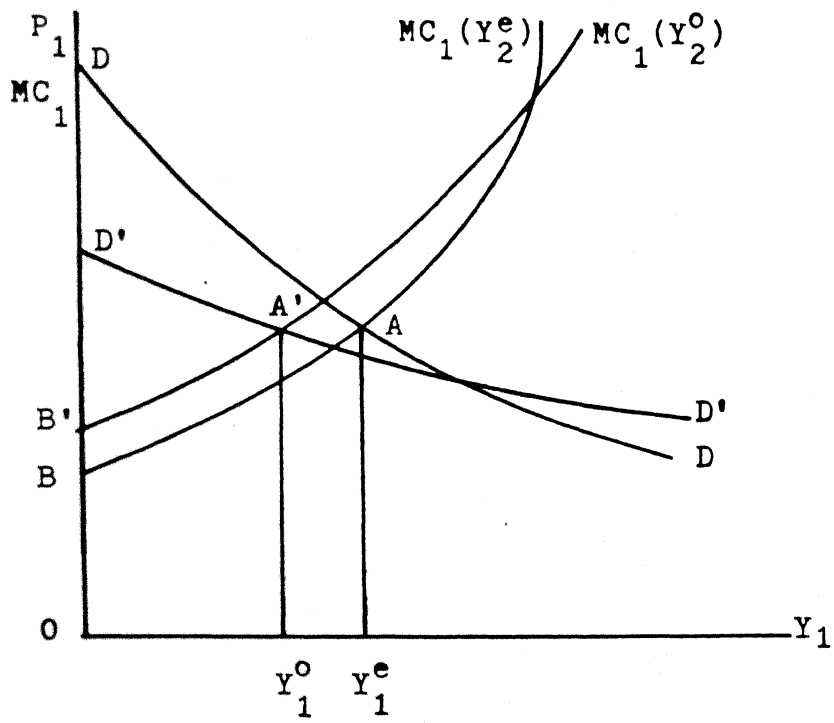


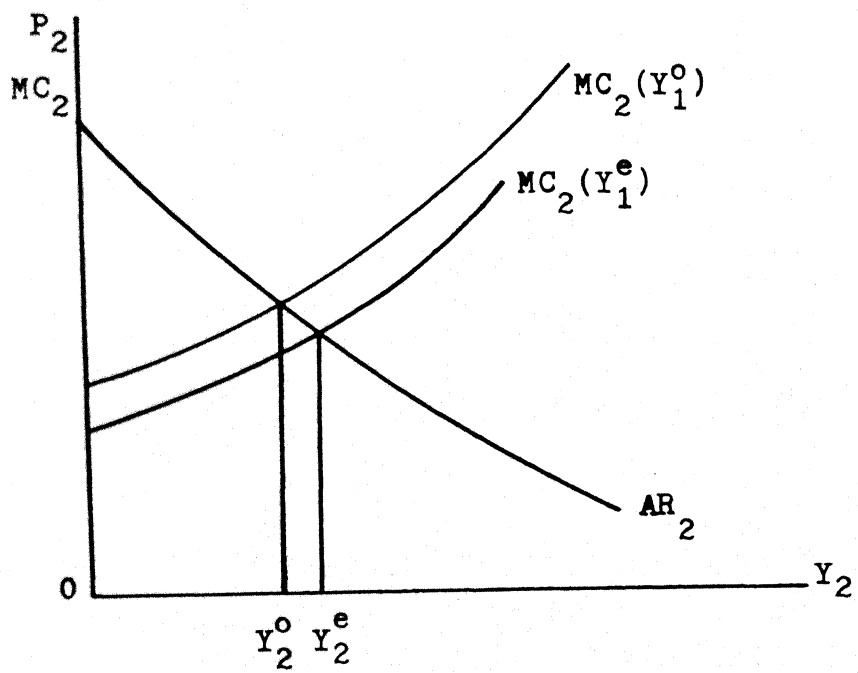
Figure 5.2.3

the demand curve for the product Y_1 in Figure 5.2.4(a) and let $MC_1(Y_2^e)$ represent the marginal cost curve for Y_1 which is made determinate by the equilibrium value Y_2^e . Y_1^e would then be the corresponding equilibrium value of Y_1 which together with Y_2^e maximizes surplus according to proposition 1 (see Chapter 4, Section 4.1). Introducing competitive changes would mean that the new demand curve is $D'D'$ in keeping with the assumptions that $f_2(\cdot) < 0$ and $f_{12}(\cdot) > 0$. The new equilibrium values of Y_1 and Y_2 , according to Proposition 6, will be both smaller than the original efficient values. Notice that with Y_2^e reduced to Y_2^o , and Y_1^e reduced to Y_1^o , the relative positions of the new marginal cost curves as compared with those pertaining to the efficient values would be in keeping with the properties of the E-boundary, and take the shapes shown in Figure 5.2.4(a). Thus, the equilibrium value of Y_1 is reduced. A similar result would hold in the Y_2 market as well. It is clear that even in these cases the total surplus is a monotonic function of the output level since previously the surplus generated in the Y_1 market was equal to the area DAB. The strong substitutability effect of competition upon the product Y_1 has reduced the total surplus generated to the smaller area D'A'B'. The diagram for the Y_2 market is represented in Figure 5.2.4(b). Notice that the indirect effect entails only changes in costs and hence Y_2 changes by a smaller amount. However, it is clear that total surplus is reduced even here.

The second part of the proposition would be related to the assumption that



(a)



(b)

Figure 5.2.4.

$$\partial^2 p_1 / \partial Y_1 \partial X = f_{12}(Y_1, X) < 0$$

From equation 5.2.5, $\frac{\partial}{\partial X} \{C_{12}(\cdot)/A(X)\} \leq 0$ as $C_{12}(\cdot) \leq 0$. This, together with equation 5.2.4, implies that the S_1 will become flatter and will always be strictly below the original level of S_1 . Therefore, it would cut the S_2 curve in the interior of the E-boundary. Again, there are direct and indirect effects of this due to cost interrelationships between Y_1 and Y_2 . Both Y_1 and Y_2 will be contracted and hence the economic reasoning is identical with the one above, except that the demand curves shift differently.

Q.E.D.

Figure 5.2.5 is representative of the contents of the proposition while Figure 5.2.6(a,b) show the relative shifts in the demand and marginal cost curves. It is evident that total surplus has gone down in these instances. However, in assessing the overall welfare effects, the surplus generated by the competitive products in question should be taken into account, and it is likely that overall welfare (including that generated by competitive entry) may go up. However, the above analysis shows that the net surplus that can be generated by this firm goes down.

CASE II : COMPLEMENTARY PRODUCTS.

It is assumed that a firm producing a product line consisting of complementary products it would operate in the region where diseconomies of scope are most likely to set in. Supposing now the firm faces competition for its principal

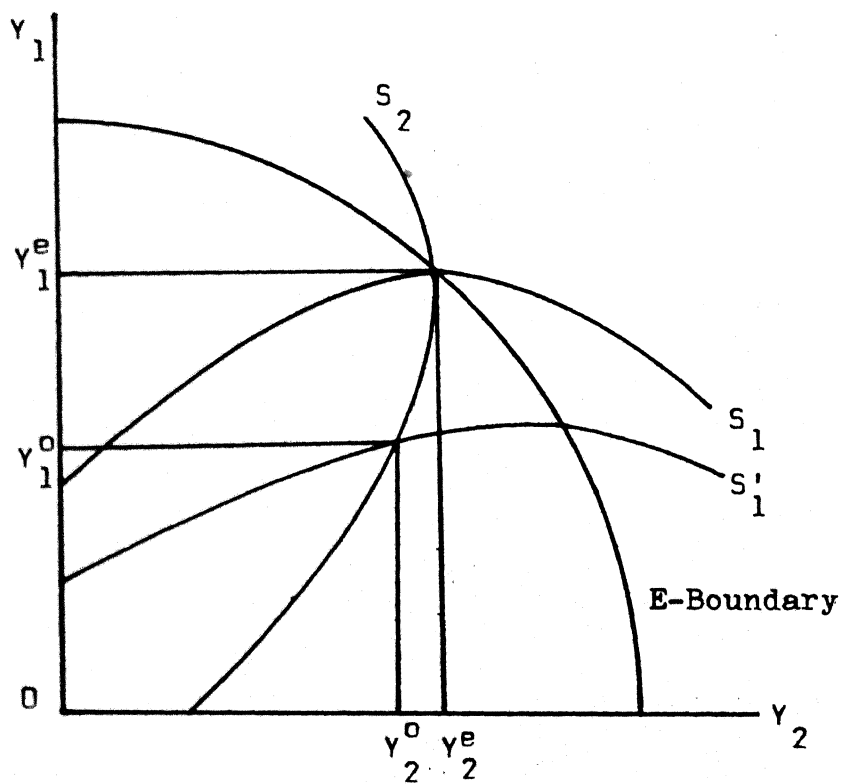
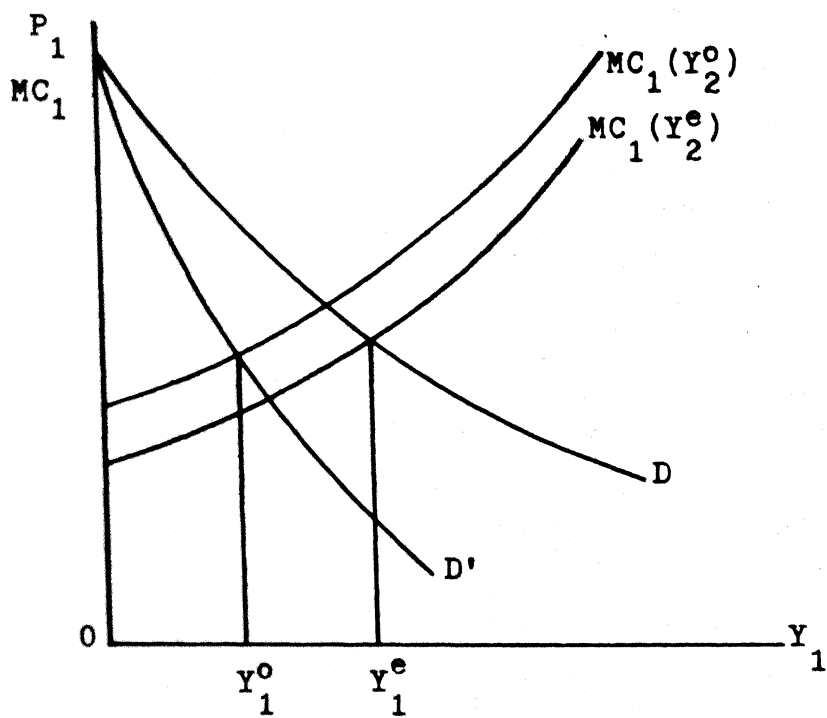
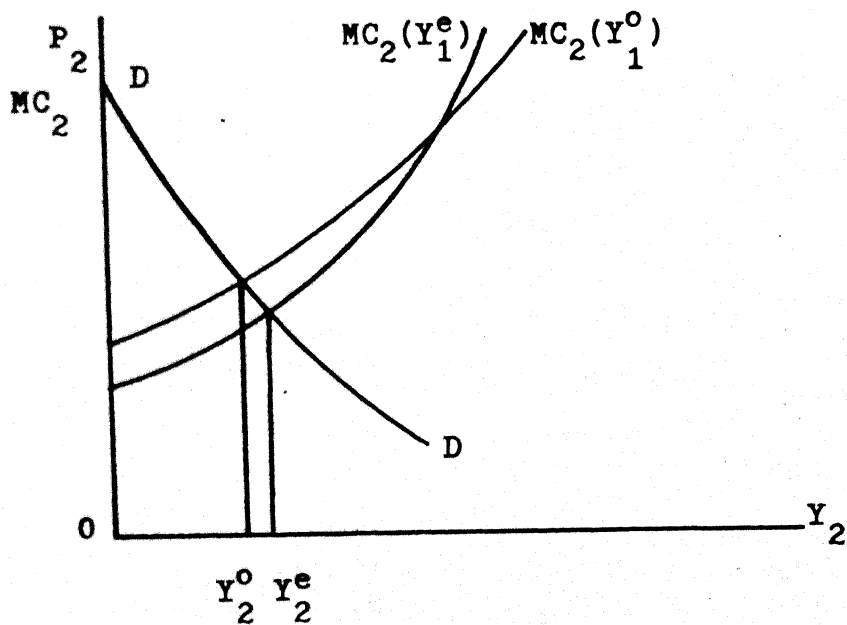


Figure 5.2.5.



(a)



(b)

Figure 5.2.6

good¹; would the effect of this competition tend to make the firm contract its output and hence that of its complementary good (thereby moving towards the E-boundary)? In other words, can market environment reduce inefficiencies within the firm? The purpose of this section is to seek answers to these questions. However, as it will be shown, the degree of reduction in the inefficiency, depends on the substitutability effects and the effect of competition on the demand function. Theoretically the number of possibilities that can emerge is rather large and since there are no a priori reasons as to which of these possibilities is likely to hold, the analysis presented here is not given the status of a proposition that can be proved.

Let $p_1 = f^1(Y_1, Y_2, X)$ be the inverse demand function for Y_1 with the properties that:

- a) $\partial p_1 / \partial Y_1 = f_1^1(\cdot) < 0$ (downward sloping demand curve)
- b) $\partial p_1 / \partial Y_2 = f_2^1(\cdot) > 0$ (Y_1 and Y_2 complementary)
- c) $\partial p_1 / \partial X = f_3^1(\cdot) < 0$ (Y_1 and X substitutable)
- d) $\partial^2 p_1 / \partial Y_1 \partial X = f_{13}^1(\cdot) \gtrless 0$ (availability of substitutes may make the demand curve steeper or flatter)

Let $p_2 = f^2(Y_1, Y_2)$ be the inverse demand function for Y_2 . The commodity X has not been included in this demand function since Y_2 need not be a proper complement of X . Such instances are not difficult to visualize. Take for instance

¹ 'Principal good' refers to the case where the products produced by the firm are not perfect complements, since perfect complements can be looked upon as a single commodity.

competition among firms producing phonograms as their principal goods with amplifiers as the complementary goods. If consumers prefer company A's product, then they would usually go in for the amplifiers of the same company, rather than buy company B's complementary item. Similarly, we have also assumed that Y_1 and Y_2 are not perfect complements in the sense that Y_2 may have some positive demand for its own sake regardless of the demand for Y_1 . However, this is not to say that there is demand independence between Y_1 and Y_2 .

Complementary demands would mean that

$$e) \quad \partial p_2 / \partial Y_1 = f_1^2(\cdot) > 0$$

$$f) \quad \partial p_2 / \partial Y_2 = f_2^2(\cdot) < 0 \quad (\text{downward sloping demand curve})$$

$$g) \quad \partial^2 p_1 / \partial Y_2 \partial X = f_{23}^1(\cdot) < 0$$

since an increase in X decreases Y_1 and hence would reduce the complementary relationship between Y_1 and Y_2 .

Let $C(Y_1, Y_2)$ be representative of the cost function whose properties have already been elaborated in Chapter 3, Section 3.3.

The total surplus is defined for the firm in question for some exogenously given level of the competitive product whose quantity is denoted by X .

$$(5.2.7) \quad S(Y_1, Y_2, X) = \int_L \left\{ \sum_{i=1}^2 (f^i(Y_1, Y_2, X) dY_i) \right\} - C(Y_1, Y_2)$$

Assuming integrability conditions and maximizing S yields the first order conditions:

$$(5.2.8) \quad S_1 = f^1(Y_1, Y_2, X) - C_1(Y_1, Y_2) = 0$$

$$(5.2.9) \quad S_2 = f^2(Y_1, Y_2) - C_2(Y_1, Y_2) = 0$$

Assuming that the second order conditions are satisfied, we have

$$dS_1 = [f_1^1(\cdot) - C_{11}(\cdot)]dY_1 + [f_2^1(\cdot) - C_{12}(\cdot)]dY_2 = 0$$

$$(5.2.10) \quad \left(\frac{dY_1}{dY_2}\right)_{S_1} = - \frac{[f_2^1(\cdot) - C_{12}(\cdot)]}{[f_1^1(\cdot) - C_{11}(\cdot)]}$$

Notice that equation 5.2.9 is the same as equation 4.1.13 and 5.2.10 is the same as equation 4.1.18 with $f_2^1(\cdot) > 0$ and being a function of X additionally. Similarly,

$$(5.2.11) \quad \left(\frac{dY_1}{dY_2}\right)_{S_2} = - \frac{[f_2^2(\cdot) - C_{22}(\cdot)]}{[f_1^2(\cdot) - C_{21}(\cdot)]}$$

which is the same as 4.1.19 with $f_1^2(\cdot) > 0$. Next observe that when $X = 0$, $Y_2 = 0$, $f^1(Y_1, 0, 0) - C_1(Y_1, 0) = 0$ is the equilibrium condition. For small changes in X ,

$$(5.2.12) \quad \left(\frac{dY_1}{dX}\right) = - f_3^1(\cdot) / (f_1^1(\cdot) - C_{11}(\cdot)) < 0$$

since both the numerator and the denominator are negative. This shows that the choice of Y_1 when $Y_2 = 0$ is smaller for a given positive value of X . Since 5.2.11 does not have X as one of its arguments the changes in the slope of only equation 5.2.10 will be examined again

$$(5.2.13) \quad \frac{\partial}{\partial X} \left(\frac{dY_1}{dY_2}\right) = - \frac{[f_1^1(\cdot) - C_{11}(\cdot)] f_{23}^1(\cdot) - [f_2^1(\cdot) - C_{12}(\cdot)] f_{13}^1(\cdot)}{[f_1^1(\cdot) - C_{11}(\cdot)]^2}$$

For simplicity let $[f_1^1(\cdot) - C_{11}(\cdot)] = A$

$$[f_2^1(\cdot) - C_{12}(\cdot)] = B$$

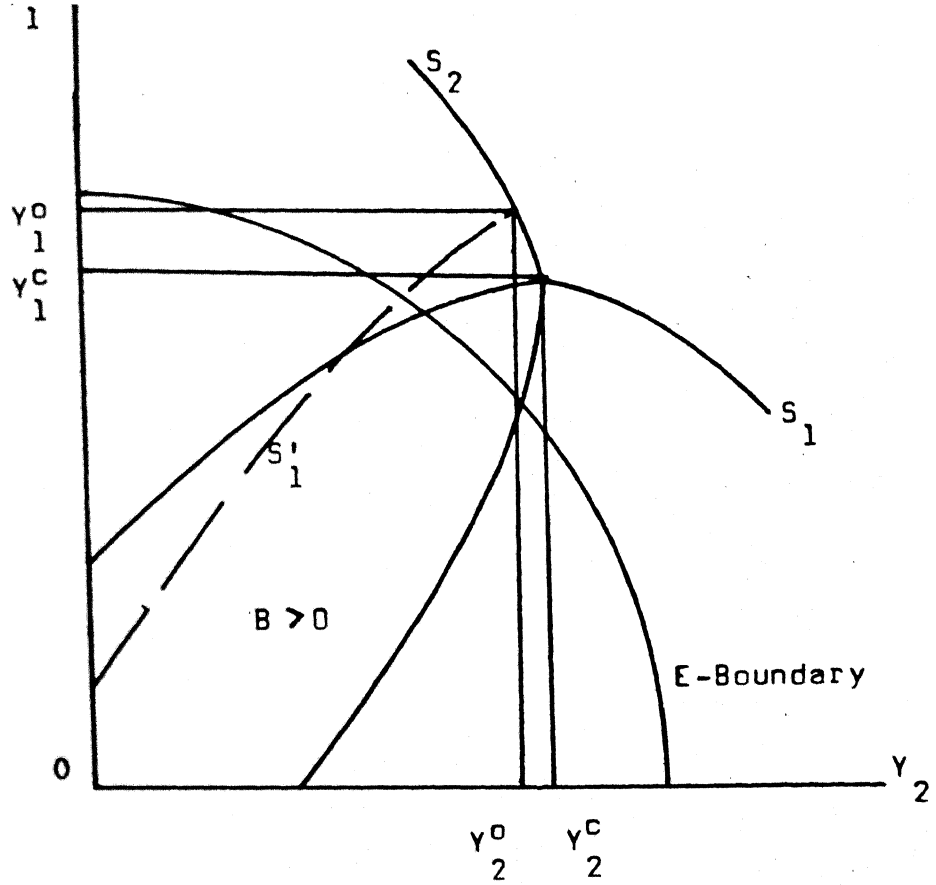
$$\text{so that } [A f_{23}^1(\cdot) - B f_{13}^1(\cdot)] = D$$

It is known that B is positive initially, zero, and negative thereafter (see Proposition 3, Chapter 4, Section 4.1, Case III). Given the assumptions on the demand curves, it remains to ascertain the sign on 5.2.13.

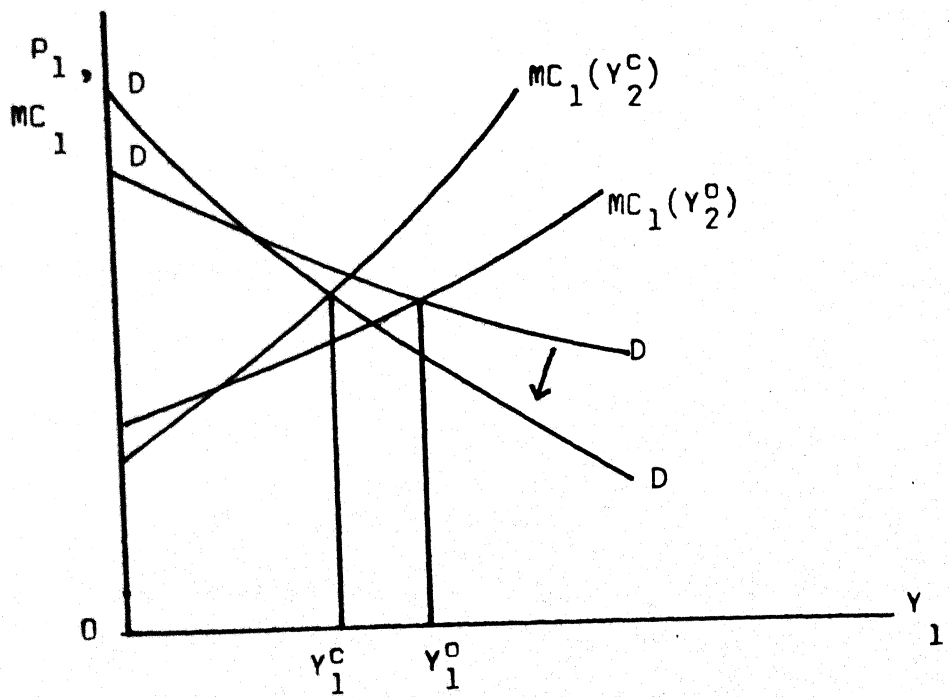
(i) Let it be assumed that $f_{13}^1(\cdot) > 0$, and $f_{23}^1(\cdot) < 0$. Since the S_2 curve does not change, the change in the slope of equation 5.2.10 is relevant only in the range $B > 0$. It is clear that, $A f_{23}^1(\cdot) > 0$ and $B f_{13}^1(\cdot) > 0$. There are three possibilities.

$$(a) \quad A f_{23}^1(\cdot) < B f_{13}^1(\cdot) \Rightarrow D < 0 \Rightarrow -D/A^2 > 0.$$

Thus the slope given by equation 5.2.10 becomes steeper, and this together with equation 5.2.13 suggests that the curve S_1 would shift in such a manner as to expand the output of Y_1 and decrease the output of Y_2 . However, this may appear unlikely since an expansion in Y_1 should lead to an increased demand for Y_2 ; but there is one crucial aspect of the analysis. Since the firm is already operating in a region of steeply rising marginal costs, a reduction in Y_2 lowers the marginal costs of Y_1 ; but, an expansion into Y_1 (which results both because of a reduction in the marginal costs as well as the demand curve being flatter) causes a sharp increase in the marginal costs of Y_2 which more than offset the increase in demand for Y_2 due to an expansion in Y_1 . Figures 5.2.7(a,b,c)

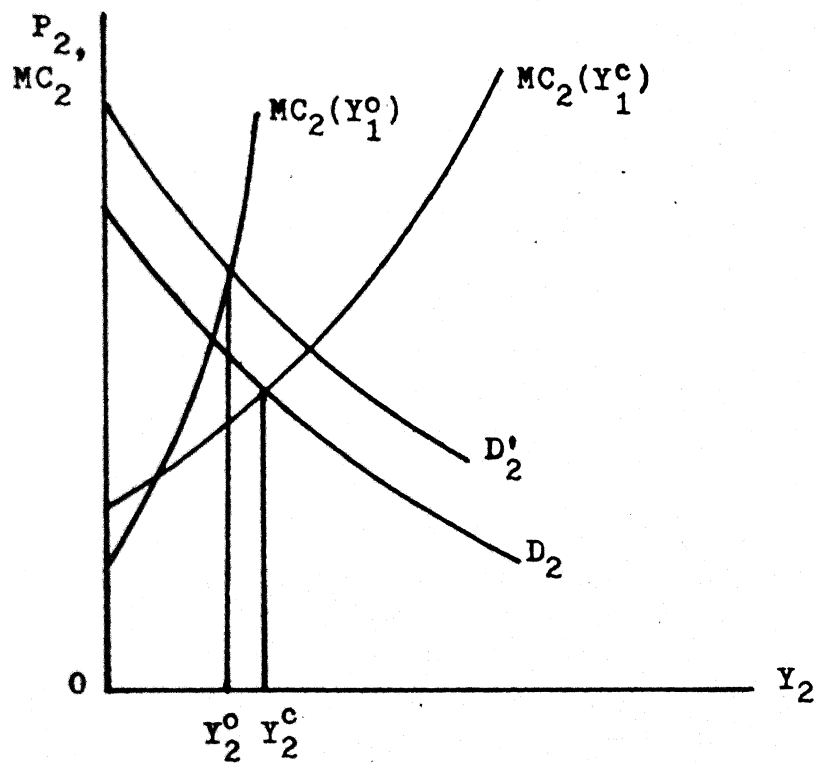


(a)



(b)

Figure 5.2.7.



(c)

Figure 5.2.7.

are representative of this argument.

$$(b) \quad A f_{23}^1(\cdot) = B f_{13}^1(\cdot) \Rightarrow D = 0 \Rightarrow -D/A^2 = 0$$

which states that there is no change in the slope of S_1 . From 5.2.13 it can be deduced that the S_1 curve shifts parallel to itself. It can immediately be concluded that, even though the point of intersection may lie above, on or below the E-boundary, the values of Y_1 and Y_2 will be lower. This is shown in the following Figures 5.2.8 (a,b,c). The arguments are analogous to the one given above. Both the above possibilities are consistent with the assumptions of demand curves and the E-boundary property of costs.

$$(c) \quad A f_{23}^1(\cdot) > B f_{13}^1(\cdot) \Rightarrow D > 0 = -D/A^2 < 0$$

Thus the slope of S_1 becomes flatter and due to 5.2.13 will be strictly below its original value. Again both Y_1 and Y_2 values will be reduced.

(ii) Let it now be assumed that $f_{13}^1(\cdot) < 0$. That is, the presence of competitive products tend to take away consumers of lower valuation of the product. $A f_{23}^1(\cdot) > 0$ and $B f_{13}^1(\cdot) < 0$ in the relevant range. Therefore, $[A f_{12}^1(\cdot) - B f_{13}^1(\cdot)] > 0$ always. Hence, $D > 0 \Rightarrow -D/A^2 < 0$. This corresponds to case (c) with the same conclusions. That is, both Y_1 and Y_2 tend to go down.

Given these possibilities, there are no a priori reasons as to which of these possibilities are likely to hold and hence the resulting ambiguity. These results, further compound the difficulty regarding what is efficient. Certainly, operating in a region of steeply increasing marginal costs is not

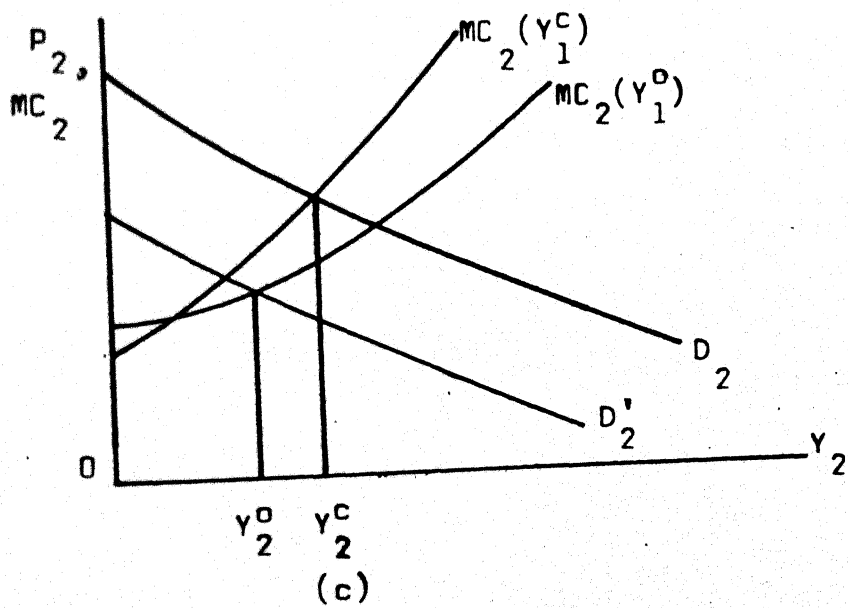
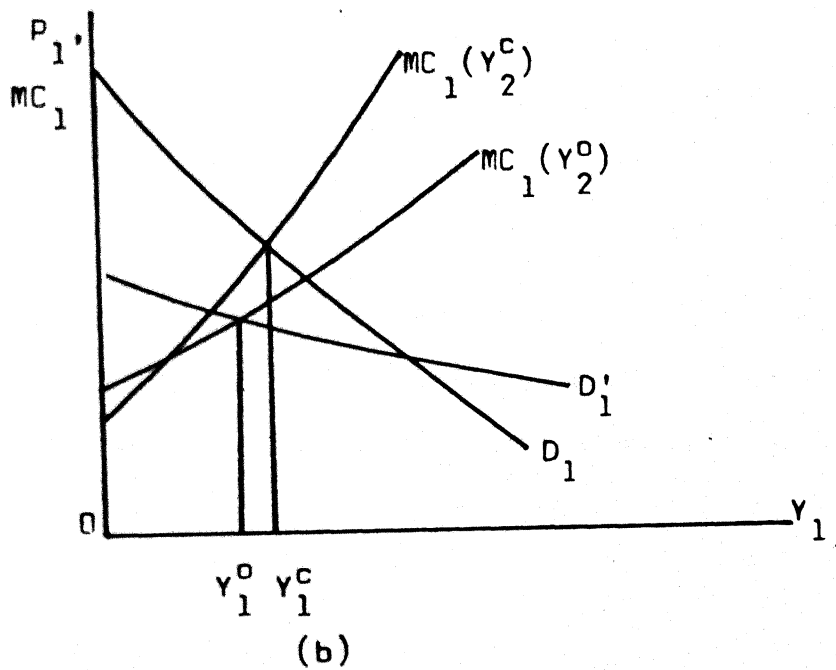
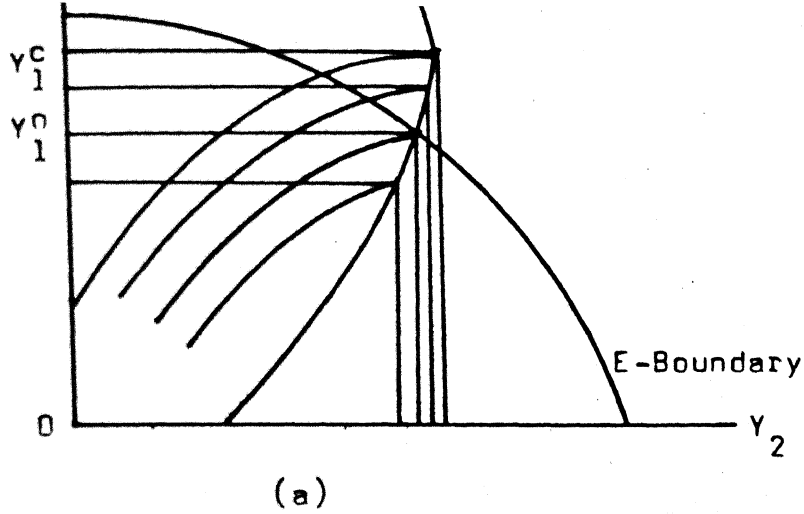


Figure 5.2.8.

cost efficient. But when looked upon from the point of view of the total surplus generated such a solution may in fact be preferred if Y_1^C and Y_2^C are greater than they were when the demand interrelationships were non-existent.

5.3 Changes in the Competitive Level and Effects on Profit Maximizing Choices

In this section two propositions are proved relating to the effects of competitive level changes on the profit maximizing levels of products which are themselves unrelated. The propositions are illustrative of the arguments already prevalent in the literature on optimal product variety. As in the previous section, the analysis is carried out using both the arguments about shifts in the demand curves. However, the marginal revenue plays an important role in ascertaining profit maximizing choices and this, in turn, is related to the elasticity of demand. It is only natural that the propositions are couched in terms of changes in elasticity of demand.

PROPOSITION 7: Let $p_1 = f^1(Y_1)$ and $p_2 = f^2(Y_2)$ be such that the profit maximizing solutions satisfy $C_{12}(\cdot) = 0$. The effect of competition on at least one of the products produced by the profit-maximizing multiproduct firm producing unrelated products will cause a deviation from the cost efficient choices whenever, the demand curve for the product is more elastic than it was previously, and the effect of outside competition on the marginal revenues of the product is positive.

Proof: Let Y_1 and Y_2 be the output of the two commodities produced by the firm and assume that they exhibit cost inter-relationships only. Let X be the level of outside competition that changes exogenously. The profit function for the firm can be written as:

$$(5.3.1) \quad \pi(Y_1, Y_2, X) \equiv Y_1 f(Y_1, X) + Y_2 g(Y_2) - C(Y_1, Y_2)$$

where $g(Y_2)$ is the demand function for the product Y_2 .

$$(5.3.2) \quad \partial \pi / \partial Y_1 = f(Y_1, X) \{1 - \frac{1}{\eta}\} - C_1(Y_1, Y_2) = 0$$

where η is the elasticity of demand for the product Y_1 . Notice that η is, in turn, a function of both Y_1 and X . Let $\eta = \eta(Y_1, X)$, further, $\partial \eta / \partial Y_1 = \eta_1 < 0$ and $\partial \eta / \partial X = \eta_2 > 0$.

When $X = 0$, the profit maximizing values for Y_1 and Y_2 are assumed to be on the E-boundary, as depicted in Figure 5.3.0a. Let Y_1^π and Y_2^π represent this solution. Since X is unrelated to Y_2 , the π_2 curve will not change. Totally differentiating equation 5.3.2, and putting $Y_2 = 0$, yields

$$(5.3.3) \quad \left(\frac{dY_1}{dX} \right)_{Y_2=0} = - \frac{[f_2(\cdot) \{1 - \frac{1}{\eta}\} + f(Y_1, X) \eta_2 / \eta^2]}{A}$$

where, $A = [f_1(\cdot) \{1 - \frac{1}{\eta}\} + f(\cdot) \eta_1 / \eta^2 - C_{11}(\cdot)] < 0$ which is the second order condition for a maximum of 5.3.1 to hold.

Let, $f_2(\cdot) \{1 - \frac{1}{\eta}\} = \theta_1 < 0$ since $f_2(\cdot) < 0$, where $f_2(\cdot) = \partial p_1 / \partial X$, $f(\cdot) \eta_2 / \eta^2 = \theta_2 > 0$ since $f(\cdot) > 0$ and $\eta_2 > 0$.

Notice that the numerator of 5.3.3 is the change in the marginal revenue of Y_1 when X changes exogenously with $Y_2 = 0$.

Equation 5.3.3 can be re-written as,

$$(5.3.4) \quad - \frac{(\theta_1 + \theta_2)}{A}$$

If $\theta_1 = \theta_2$ then equation 5.3.4 takes the value zero. That is, the value of Y_1 (for $Y_2 = 0$) does not change when X varies. This condition is shown in Figure 5.3.0b. Refer to this as "condition 1". This evaluates the intercept of π_1 when X varies exogenously. However, for any positive value of Y_2 , the MC_1 curve would be lower. Hence, the value of Y_1 would be higher. This simple intuition is enough to indicate that for given values of Y_2 , where $C_{12} < 0$, the corresponding values of Y_1 would be higher, and hence, the profit maximizing choices would be higher than when there was no competition. This can be demonstrated algebraically as follows.

The change in the slope of π_1 for changes in X is given by:

$$(5.3.5) \quad \frac{\partial}{\partial X} \left(\frac{dY_1}{dY_2} \right) = - \frac{C_{12}(\cdot) \partial A / \partial X}{A^2}$$

$$\begin{aligned} \text{evaluating } \partial A / \partial X &= [f_{12}(\cdot) \{1 - \frac{1}{\eta}\} + f_1(\cdot) \eta_1 / \eta^2 \\ &\quad + f_2(\cdot) \eta_1 / \eta^2 + f(\cdot) \partial \{\eta_1 / \eta^2\} / \partial X] \end{aligned}$$

$$\text{evaluating } \partial \{\eta_1 / \eta^2\} / \partial X = (\eta_{12} \eta^2 - \eta_1 \eta \eta_2 + \eta \eta_2) / \eta^4 > 0$$

if we assume $\eta_1 < 0$ and $\eta_{12} = 0$. Therefore,

$$(5.3.6) \quad \partial A / \partial X > 0$$

In the region $C_{12}(\cdot) < 0$, 5.3.5 is positive. Hence, the π_1 curve will now be steeper than it originally was. When

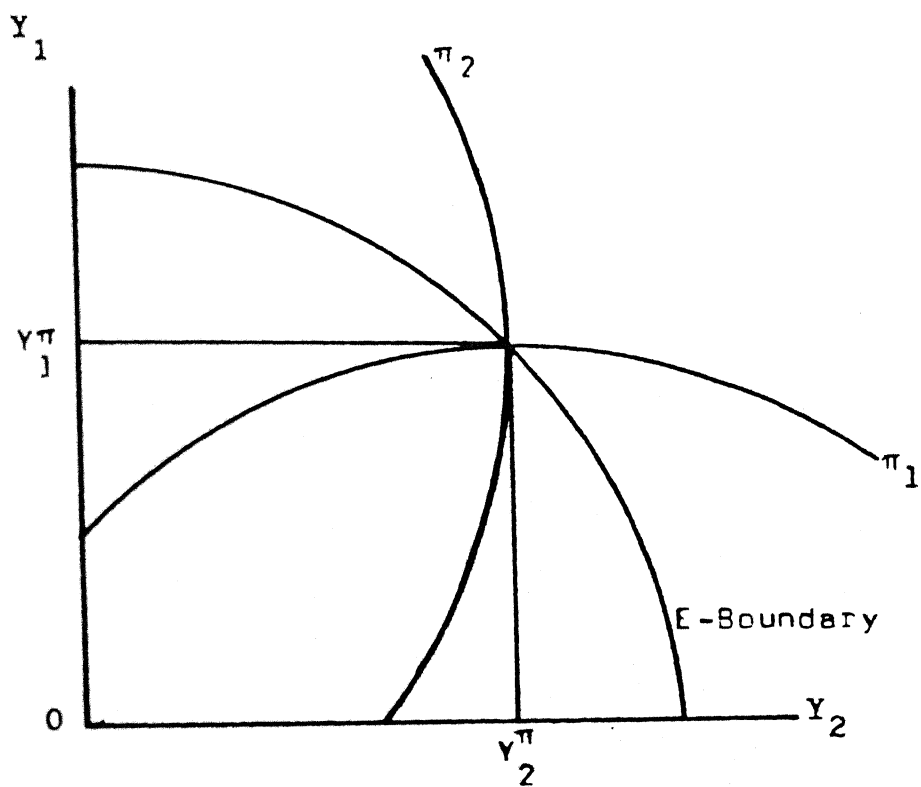
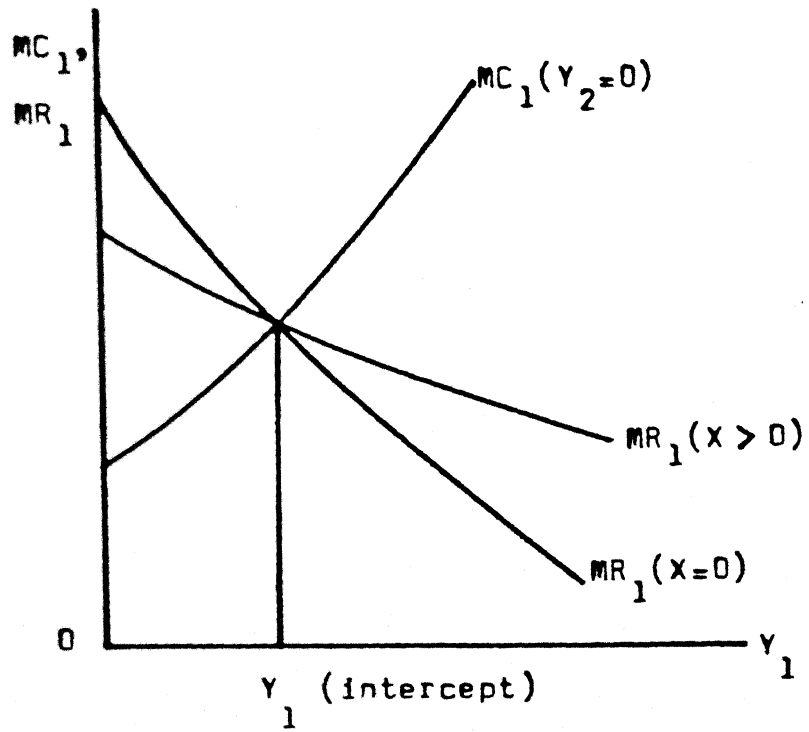


Figure 5.3.0.a



Condition I

Figure 5.3.0 b.

$C_{12}(\cdot) = 0$, equation 5.3.5 attains a zero value and when $C_{12}(\cdot) > 0$, the curve would now be steeper than it previously was. Given "condition 1" the new π_1 curve (call it π_1') would now cut the π_2 curve outside the E-boundary. Figure 5.3.1 is representative of this.

Q.E.D.

The above argument is depicted in the Figure 5.3.2(a,b) in terms of the relative shifts in the marginal cost and revenue curves. It is evident that profits would be adversely affected in the Y_2 market. However, in the Y_1 market, profits may in fact increase since the demand curve has become more elastic, and reductions in profits arising out of a cost inefficient configuration in the Y_2 market may be more than offset by gains in the Y_1 market. This explains why firms may incur higher costs in one product line and yet may do nothing to avoid this if they can cross-subsidize from other product lines.

Recall that, equation 5.3.4 shows the effect upon the marginal revenues obtainable from Y_1 when the competitive level changes. Till now it was assumed that $\theta_1 = \theta_2$. However, if $|\theta_2| < |\theta_1| \Rightarrow \theta_1 + \theta_2 < 0$ and equation 5.3.4 will be negative. Hence, the intercept of π_1 would be lower with competition. This together with equation 5.3.5 will give rise to two possibilities. The first case is depicted in Figure 5.3.3(a,b). In this case, the original solution is maintained since the producers are unable to convert more consumer surplus into profits. This in turn may make for the

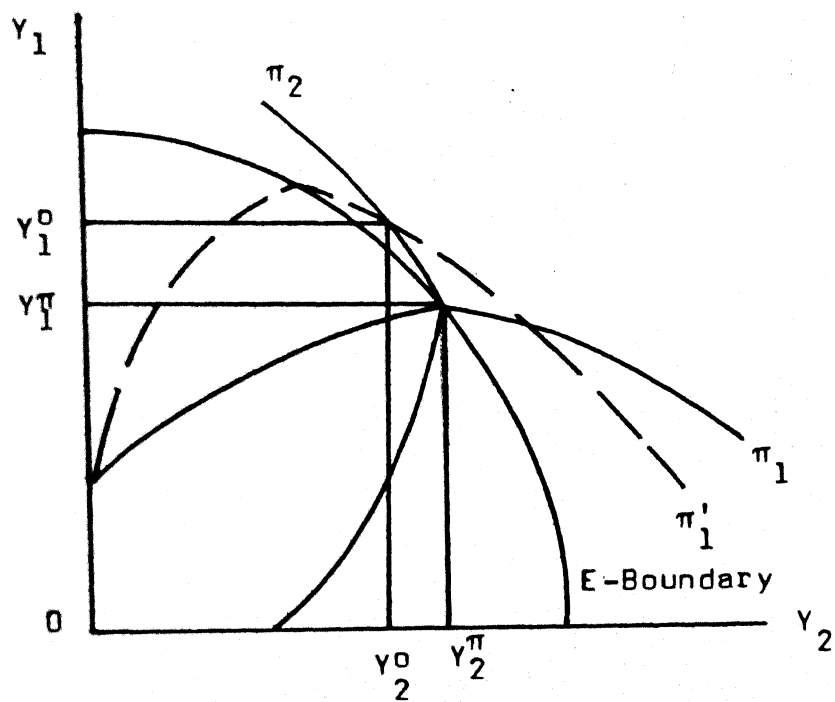
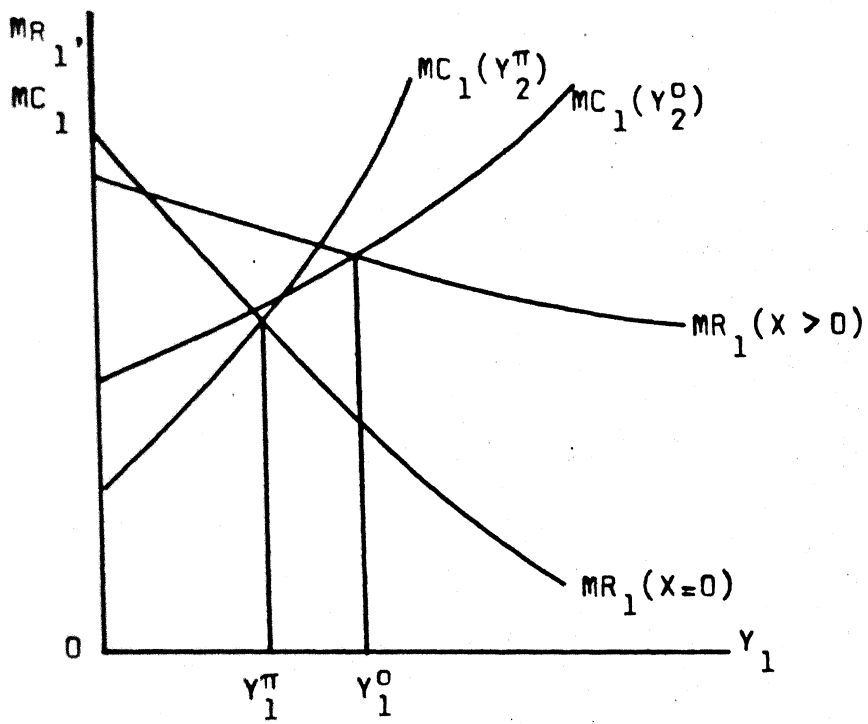
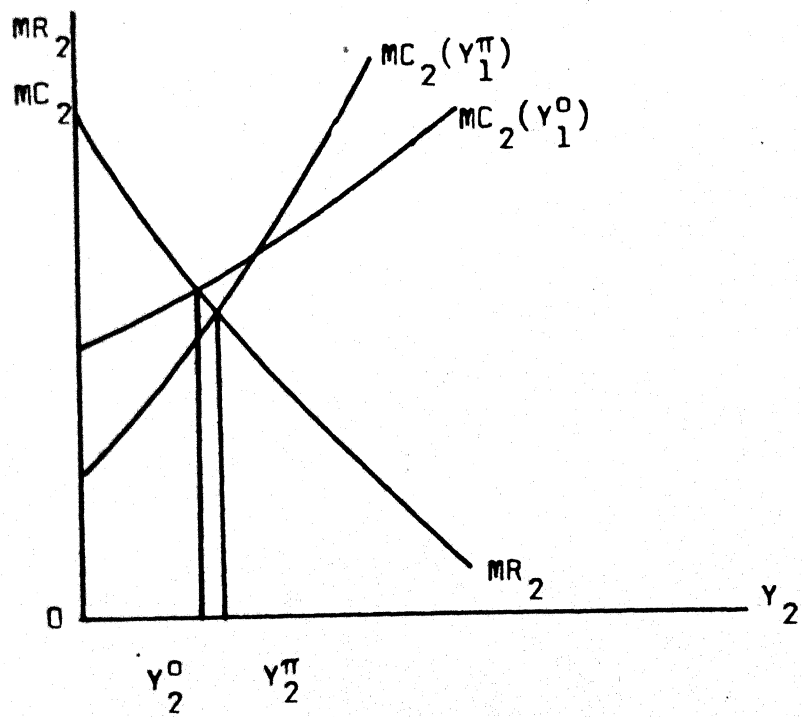


Figure 5.3.1

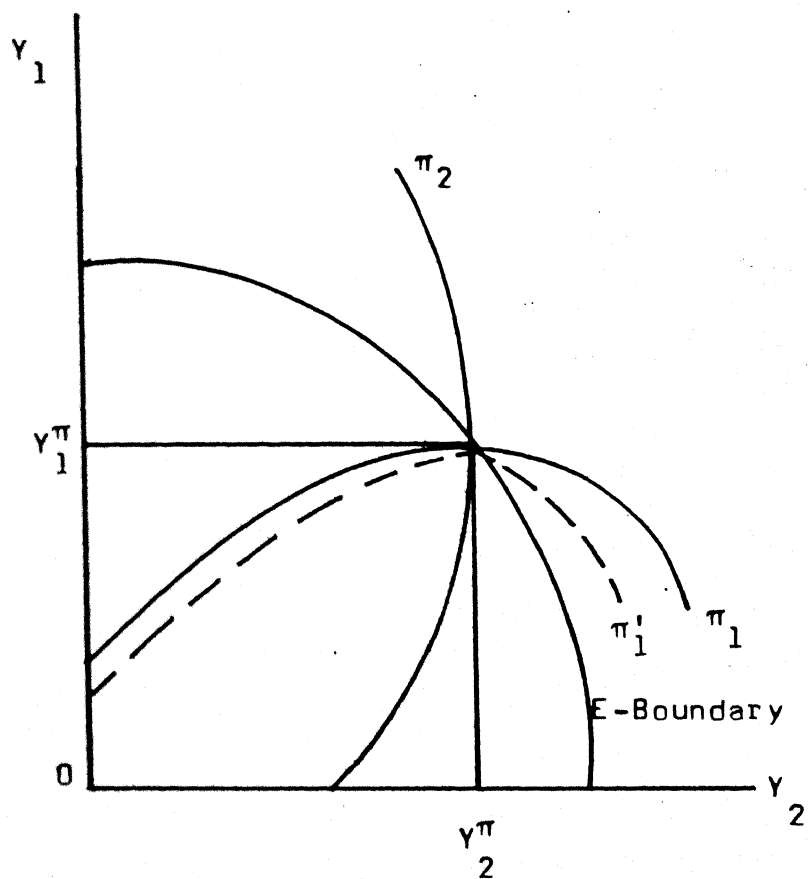


(a)

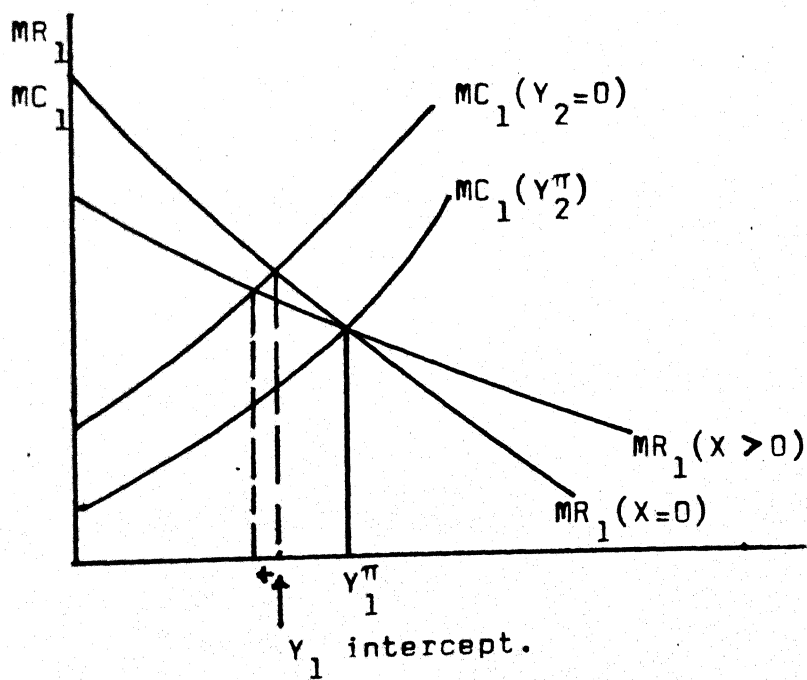


(b)

Figure 5.3.2



(a)



(b)

Figure 5.3.3.

firm dropping the product altogether. This is happening even when the elasticity of demand is changing favourably, since according to Spence (1976a) products with low elasticity of demand have difficulty surviving the market. Although it shall be shown presently that this is generally true, the present analysis shows that under certain conditions, i.e., when $|\theta_2| < |\theta_1|$ those with higher elasticity may also face a drop in profits.

The other possibility is that the π_1 cuts π_1 to the left of the original equilibrium point and hence cuts π_2 outside the E-boundary as shown in Figure 5.3.1. The arguments would be analogous to those represented by Figures 5.3.1 and 5.3.2(a,b) and hence are not repeated here. Once again inefficiency results. To complete the analysis, it is necessary to examine those cases wherein the demand curve for the product becomes less elastic. It is again assumed that the demand curves are so as to allow the profit maximisation solution to be where $C_{12}(.) = 0$ when there is no competition.

PROPOSITION 8: The effect of competition on at least one of the products produced by the multiproduct-profit-maximizing firm producing unrelated products will cause a deviation from the cost efficient choices of output levels whenever,

(i) The demand curve for the product becomes less elastic,

(ii) The infra-marginal revenues obtainable from the product in question are adversely affected by competitive forces.

Proofs: In this case, $\eta_2 < 0$. From equation 5.3.3 it can be deduced that the quantity of Y_1 for $Y_2 = 0$ would in fact be less than what it would have been had there been no change in competition. Now, $\theta_2 < 0$, and $\partial A/\partial X$ represent the effect on infra-marginal revenues obtainable from Y_1 as a result of changes in X . If these were adversely affected, $\partial A/\partial X < 0$. From equation 5.3.5, in the region $C_{12}(\cdot) < 0$, the sign on 5.3.5 is negative. Given this and $\eta_2 < 0$ establishes the proposition since π'_1 would be strictly below π_1 and hence would cut π_2 in the interior of the E-boundary. Both Y_1 and, to a smaller extent Y_2 , would come down resulting in lower profits in both the markets. This is shown in Figure 5.3.4.

Q.E.D.

5.4 Some Observations

Several pertinent observations are in order.

A) Spence (1976a), in dealing with product selection under monopolistic competition, makes the observation that the products with lower elasticities of demand have difficulty surviving the market. If η be the elasticity of demand for a product then the ratio β of gross revenues to gross-surplus, is given by the relation $\beta = (1 - \frac{1}{\eta})$, since,

$$\beta = Y f(Y) / \int_0^Y f(\theta) d\theta$$

$$\eta = - \frac{dY}{dp} \cdot \frac{p}{Y} \quad \text{or,} \quad \eta = - \frac{p}{f'(Y)Y}$$

$$p = -\eta f'(Y)Y. \quad \text{Let} \quad I = \int_0^Y f(\theta) d\theta$$

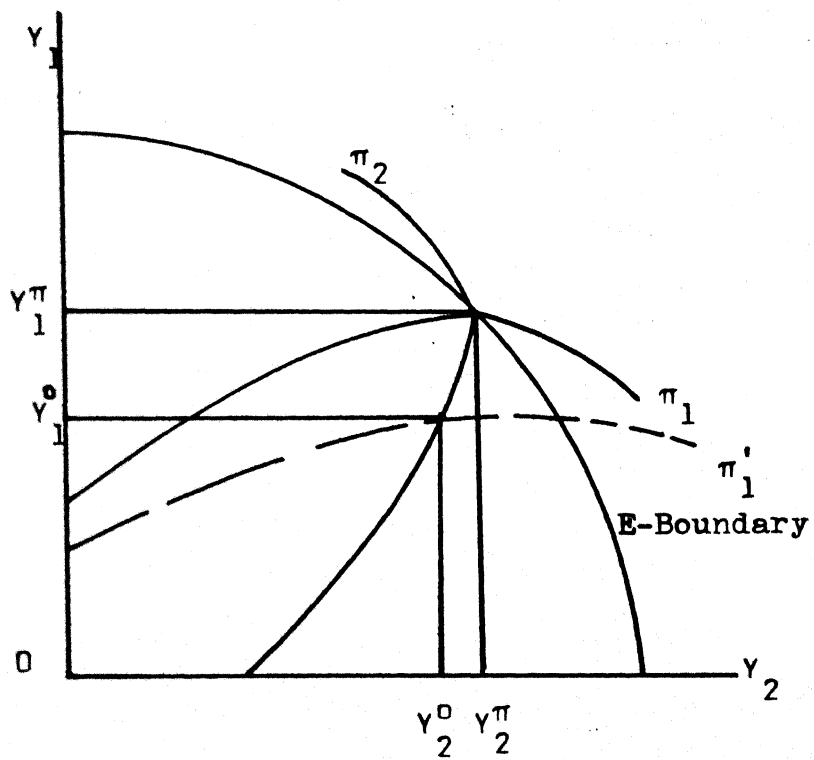


Figure 5.3.4.

$$I = -\eta \int_0^Y \theta f'(\theta) d\theta = -\eta [Y f(Y)]_0^Y - \left\{ \int_0^Y f(\theta) d\theta \right\} \eta$$

or,
$$I = -\eta [Y f(Y) - I].$$

therefore,
$$I \{1 - \eta\} = -\eta Y f(Y)$$

or,
$$I \left\{ -\frac{1}{\eta} + 1 \right\} = Y f(Y)$$

but,
$$I = \int_0^Y f(\theta) d\theta$$

therefore,
$$I = Y f(Y) \frac{1}{1 - \frac{1}{\eta}}$$

or
$$\beta = \frac{Y f(Y)}{I} = 1 - \frac{1}{\eta}.$$

Therefore, as η increases the ratio β rises. If η decreases then this ratio goes down and hence profits of the firm are adversely affected. This would be a force which may make for the non-production of such goods. Our results for the effect of competition requires that β decreases as one of the conditions leading to contraction in output since,

$$\partial \beta / \partial X = \eta_2 / \eta^2 < 0 \quad \text{iff} \quad \eta_2 < 0.$$

The other condition is that the infra-marginal revenues obtainable should go down. If this condition is not met, then the possibility of the firm retaining its output or cross-subsidize losses in one product with gains in another is not ruled out. Such possibilities were, of course, not open in the framework of Spence since he assumed that each firm produced only one product.

B) Useful insights can be gained from the concept of 'survival ratio' (Spence 1976b) when applied to the present analysis. To demonstrate this, assume a profit-maximizing firm producing two products (unrelated in demand) Y_1, Y_2 having significant cost interrelationships. The 'survival factor' is simply the ratio of total revenues to total costs attributable to each product that the firm produces:

$$\rho_i = p_i f(Y_i) / \int_0^{Y_i} C_i(\theta, Y_j) d\theta$$

$$i \neq j; i, j = 1, 2$$

The product would survive if,

$$\int_0^{Y_i} C_i(\theta, Y_j) d\theta \leq p_i f(Y_i)$$

In ~~proposition~~^{chapter} 4, it was ~~proved~~^{argued} that the profit-maximizing quantities Y_1 and Y_2 ~~were~~^{can be} so as to fulfil the condition that $C_{12}(\cdot) = C_{21}(\cdot) = 0$ meaning that the marginal costs of production of each of the outputs were at their lowest possible levels. Therefore, for any other value of $Y_i \neq Y_i^\pi$, $Y_j \neq Y_j^\pi$, would mean

$$C_1(Y_i^\pi, Y_j^\pi) > C_1(Y_i, Y_j^\pi)$$

and $C_2(Y_i^\pi, Y_j^\pi) > C_2(Y_i, Y_j^\pi)$.

Now assuming the conditions postulated in this chapter, proposition 8 would mean: (i) marginal costs are not minimized, and (ii) lower total revenues.

Lower total revenues, in this case arise partly from the higher costs incurred in not being able to exploit cost

complementarities (as of economies of scope). Therefore, p_i falls. It would fall faster for that product which faces competition, and it would fall slower for that product whose marginal costs are rising only. Indeed, if the firms were to select products keeping in view the market competition that is anticipated, the products may not be produced at all. However, the opportunity of exploiting economies of scope would also play an important role in the selection of the firm's products, since, even though the firm is unable to exploit this completely, it may still be at an advantage vis-a-vis single product firms. The dual notions of maximizing efficiency (total surplus) and profits would be compatible only if the existing demand conditions allow it to operate on the E-boundary.

This completes the analysis of horizontally integrated firms. The next chapter deals with the efficiency of yet another form of multiproduct production, that is, the vertically integrated firm.

CHAPTER 6

EFFICIENCY OF BACKWARD VERTICAL INTEGRATION WITH TRANSACTION COSTS

6.1 Introduction

In Chapter One it was argued that multiproduct production can take a variety of forms such as the production of related goods or that of unrelated goods. From the point of view of costs, it was argued that whenever the production of such goods offers economies of scope, reflected in the economies obtainable by producing the goods together (rather than separately, each by its own specialized firm), multiproduct production will emerge and be more efficient.

On the other hand, it has been acknowledged that vertically integrated firms are also instances of multiproduct production and hence, it would be worthwhile to examine whether the concept of cost efficient choices of output levels developed in the previous chapters would extend to the case of a vertically integrated firm. Vertical integration, in turn, can range from a upstream firm integrating into downstream production activities (Forward integration) or a firm integrating into the production of inputs (Backward vertical integration). Yet another form of vertical integration is that a firm simply acquires the input suppliers. It is sufficient to note for the present study that any cost advantages of vertical integration that result in lower end product prices and/or the generation of a greater total surplus than

those of the non-integrated counterparts would be sufficient to consider vertical integration as being efficient.¹

Of the various reasons why a firm would undertake vertical integration, expected savings in costs is a common reason whether it be forward vertical integration (Needham 1969; Singer 1968), or backward vertical integration.² It has been recognized that one such advantage could be the saving of costs of going to the market. Such costs are referred to as 'Transaction costs' in the literature and, as Williamson (1971) points out, it is one of the main reasons for firms to undertake backward vertical integration. A brief resume of the 'Transaction costs' argument is called for before proceeding further.

6.2 Transaction Costs and Vertical Integration

Perhaps the best way to highlight the role of transaction costs is to argue along the lines of Williamson (1971).³

¹The more difficult problem of analyzing vertical integration as having antisocial and anticompetitive properties is beyond the scope of this study.

²This is evident from the arguments of the Chicago school in general and from the work of Perry (1978) and Scherer (1980) in particular. Specific conditions of cost reductions when input-substitution is possible has been examined by Mallela and Nahata (1980).

³The basic proposition that firms exist for purely economic reasons (Coase 1937) has led to many contributions on the inherent benefits of the suppression of the price mechanism, especially those of Alchian and Demsetz (1972), Klein, Crawford, and Alchian (1978). However, Williamson's analysis is based directly upon the insights from Coase's paper and is representative of the existing literature in this area.

Consider a firm engaged in the production of a final commodity which buys the required inputs on the market. If the market for one such input is imperfect, a monopolistic price over cost is commonly anticipated under the circumstances. A once-for-all contract might be negotiated with the input supplier by the final goods firm. In a perfectly static environment, integration holds no advantage over once-for-all contracts. If, however, a sort of bilateral-oligopoly emerges the range of bargaining alternatives is broadened and the expected bargaining or haggling would constitute a drain on the joint profits and would also be socially unproductive. Thus an incentive to avoid some of these 'costs' may emerge. One possible way is through vertical integration or by the negotiation of a once-for-all contract. However, if the nature of the final good is such that periodic changes in volume and design are called for, then once-for-all contracting may not be the best policy. Of the two alternatives available viz., short-term contracting and vertical integration, the former would pose problems of costly recontracting procedures for the firm and a disincentive for long term investment considerations by the input supplier. Hence, efficient supply of the input may suffer. This, in turn, will add to the costs of the final goods industry. Such transaction costs may be avoided by vertical integration.

Almost all contracting procedures have an element of trust built into them. This, in turn, would depend upon the information available to the contracting parties about each other. The perceived risks of the two parties, under a lack

of information may make it difficult to negotiate a contract that offers commensurate returns to each because of their inability to reveal that they are 'good' risks. Such information 'costs' which are responsible for higher costs of transactions on the market can be avoided by vertical integration. In other words, the objective risks arising out of imperfect information are augmented by contractual risks and integration amounts to self-insurance by those individuals, or the firm to be good risks. Thus while there are costs of writing contracts which, are the end products of negotiations, there are also costs involved in the enforcement of contracts. In the words of Hess (1983, p. 36): "Negotiation costs are those resources used up in the process of arranging mutually satisfactory exchanges of property, while enforcement costs are resources used up with the aim of preventing breach of contractual stipulations".

To summarize, transaction costs are those resources used up in the drawing up of a contract between two parties, and the subsequent enforcement of the same. Thus transaction costs may involve bringing agents together (costs of communication), acquiring and disseminating information about the terms of exchange (information costs) drawing up of contracts that are legally binding and the enforcement of such contracts. The costs associated with once-for-all contracts would take the form of fixed costs of drawing up of the contract. However, there may still be a variable cost component of enforcement of the same, whereas, if the process of re-contracting is needed, then it would be a variable cost in itself.

De Alessi (1981) assumes that the larger is the volume of transaction to be conducted the greater is the transaction cost involved.

From the point of view of vertical integration, for the same transactions, a shift towards a vertically integrated governance structure would permit a simultaneous reduction in both the expenses of writing complex recurrent contracts and the expenses of executing it effectively in an adaptive, sequential way.⁴ In the present study, the cost-advantages obtainable from such reductions in transaction costs are recognized. At the same time it is also acknowledged that costs of using the market is not limited to transaction costs. There may well be other costs such as transportation of the raw materials (freight charges), costs of 'searching' for the right prices, costs involved in paying above the marginal costs of production of the input purchased and costs involved in taking delivery of the inputs purchased.⁵ It is of interest to note that the avoidance of transaction costs, while offering advantages to internalizing transactions within a single enterprise, would account for the notion of 'firm subadditivity' developed by Sharkey (1982).

On the other hand, it has also been recognized that organizing an economic activity into what is commonly referred

⁴For detailed discussions on transaction costs of this kind see Williamson (1971, 1979).

⁵It can be equally well argued that all these costs are already taken care of when a contractual agreement is reached, and hence, are included as transaction costs.

to as 'the firm' is itself not costless. Coase (1937) suggests that a firm will expand until the marginal value of making an additional transaction internal to the firm equals the cost at the margin of market transactions. These costs are in turn related to the fixed supply of entrepreneurial input. In general terms, the organization of economic activity within the enterprise would mean that there would be costs of coordinating, monitoring and supervising activities within the firm (intra-firm transaction costs) which are quite distinct from the costs of operations which are purely technological in character. These costs, in themselves, could put a limit to the internalization process that is envisaged at any point of time. To the extent that the Penrosian arguments of emerging managerial excess capacity over time are accepted, the ability of the firms to expand by internalizing is enhanced. Whether a set of transactions ought to be executed across markets or within a firm would depend upon the relative efficiency of each mode. Vertical integration would emerge as a more efficient mode of input procurement if the set of transactions performed across the market is more costly than within the firm.

6.3 Characterization of Vertical Integration

The following model deals with the case of backward vertical integration. The many models of vertical integration, studied in the past (Perry 1978; Carlton 1979; Greenhut and Ohta 1979; Mallela and Nahata 1980; Dixit 1983; Vernon and Graham 1971, Warren-Boulton 1974 etc.) have all ignored

6.4 The Model: Preliminary Considerations and Assumptions

Consider a firm producing an output Y using certain inputs. Let X represent one such input that the firm buys on the market for use in the production of Y . It is assumed that,

(A) (i) X is sold in a monopolistically competitive market.

(ii) The prices of the other inputs used by the final goods firm are assumed to be fixed and,

(iii) These inputs are not substitutes for X , in the production technology of Y .

Let $X = g(Y)$ represent the input requirement function for Y units of output.

Let $p(X)$ be the price paid for the input on the market. It is generally accepted that when the input market is not competitive larger amounts of the input can be obtained at higher prices.

Let $T(X)$ represent the costs of using the market. These would be basically the transaction costs incurred in buying X units on the market plus other costs such as search costs, and the costs of transportation, etc. Of these, the costs of transacting are given a primary place in this study. The following assumptions are made:

$$(B) \quad (i) \quad \partial X / \partial Y = \partial g(Y) / \partial Y = g_1(Y) > 0$$

(The marginal requirements of input X are positive.)

$$(ii) \quad \partial g_1(Y) / \partial Y = g_{11}(Y) > 0$$

(Increasing input requirement for larger volumes of output.)

$$(iii) \quad \partial T(X)/\partial X = T'(X) > 0$$

$$(iv) \quad \partial T'(X)/\partial X = T''(X) = 0$$

(Transaction costs increase linearly at the margin.

The assumption of linearity is a matter of simplification only.

It could very well be increasing at an increasing rate, but this does not affect the generality of the results.)

$$(v) \quad \partial p(X)/\partial X = p'(X) > 0$$

(This is a consequence of assuming an imperfectly competitive market for the input.)

The total cost function for a non-integrated firm producing Y can be written down as

$$(6.4.1) \quad C(Y) = \bar{C}(Y) + [p(X)X + T(X)]$$

if Y units of output are produced. $\bar{C}(Y)$ would represent the costs of producing Y which include the fixed and variable costs but exclude the costs of buying the input on the market. More generally, $\bar{C}(Y)$ would represent both the technically determined costs of production of Y as well as the intra-firm transaction costs, given a size of plant and organizational structure.

It is quite obvious from this formulation that savings in the price-cost margin that has to be paid to the monopolistic supplier of the input X , and the associated transaction costs of a given volume of input purchased are two factors which may motivate the firm to vertically integrate into the production of X . This is of course the second component of the total-cost function 6.4.1. Let

$$(6.4.2) \quad p(X)X + T(X) = \Phi(X)$$

$\Phi(\cdot)$ is the 'expenditure function' associated with the purchase of the input X . If I represents the amount of input produced by the integrated firm, then it is evident that, given a value of Y , a firm which is

- (i) Fully vertically integrated would mean, $I = g(Y)$
- (ii) Partially vertically integrated implies $I < g(Y)$
- (iii) More than fully integrated implies $I > g(Y)$.

Equivalently, if $Z = (g(Y) - I)$, full vertical integration would mean $Z = 0$, partial integration is when $Z > 0$ and more than full integration implies $Z < 0$. Reconsider the expenditure function 6.4.2,

$$E = \Phi(X) = \Phi(g(Y))$$

when an amount I of input is being produced by the integrated firm,

$$(6.4.3) \quad E = \Phi(g(Y) - I) = \Phi(Z)$$

would be the **total** market expenditure. Thus, the total cost function for an integrated firm is

$$(6.4.4) \quad C(Y, I) = \bar{C}(Y, I) + \Phi(Z)$$

such that i) $\partial C(\cdot)/\partial Y = C_1(\cdot) > 0$

ii) $\partial^2 C(\cdot)/\partial Y^2 = C_{11}(\cdot) > 0$

iii) $\partial \bar{C}(\cdot)/\partial Y = \bar{C}_1(\cdot) > 0$

iv) $\partial^2 \bar{C}(\cdot)/\partial Y^2 = \bar{C}_{11}(\cdot) > 0$

$$v). \partial \bar{C}(\cdot) / \partial I = \bar{C}_2(\cdot) > 0$$

$$vi) \partial^2 \bar{C}(\cdot) / \partial I^2 = \bar{C}_{22}(\cdot) > 0$$

The function $\bar{C}(\cdot)$ would now also include the costs of production associated with the input, and the firm specific transaction costs of internalizing this additional activity. The technological and organizational characterization of producing Y and I jointly can take any one of the possibilities described in Section 5.3. When there are no economies of joint production, the function $\bar{C}(Y, I)$ would have the property that,

$$\partial^2 \bar{C}(Y, I) / \partial Y \partial I > 0$$

when economies of joint production prevail then,

$$\partial^2 \bar{C}(Y, I) / \partial Y \partial I < 0$$

and the range of the possible (Y, I) values over which this will hold will be delimited by the E-boundary defined in Chapter Three.

The behaviour of the total cost function depends upon the behaviour of the expenditure function for changes in Y and I as well. It would be convenient to list some of the properties of the expenditure function at this stage. Since,

$$\Phi(Z) = p(Z)Z + T(Z) \quad \text{where } Z = (g(Y) - I)$$

$$(C) \quad (i) \quad \text{For } I = g(Y), Z = 0, \Phi(0) = 0$$

$$(ii) \quad I > g(Y), \bar{Z} < 0, \Phi(\bar{Z}) < 0$$

implying a revenue obtainable by selling input on the market.

Notice that at $I > g(Y)$, $|\bar{Z}| = \bar{Z} p(\bar{Z})$ only and $T(\bar{Z}) = 0$.

$$(iii) \quad I < g(Y), Z > 0, \Phi(Z) > 0$$

Given the assumptions (B i-v), the following properties of $\Phi(Z)$ can be shown to hold.

$$(6.4.5) \quad \partial \Phi(Z) / \partial Z = \Phi_1(Z) > 0$$

since $\Phi_1(Z) = p(Z) + Z p'(Z) + T'(Z) > 0$. That is to say, the greater is the volume of input purchased on the market, the larger would be the expenditure incurred. This is expected to rise at an increasing rate, since

$$(6.4.6) \quad \partial^2 \Phi(Z) / \partial Z^2 = \Phi_{11}(Z) > 0$$

since $\Phi_{11}(Z) = 2p'(Z) + p''(Z)Z > 0$ (even if $p''(Z) = 0$).

The change in the expenditure function for changes in Y will be expected to behave in a similar way since $g_1(Y) > 0$ which directly has a bearing on the $\Phi(\cdot)$ function. Thus,

$$(6.4.7) \quad \partial \Phi(Z) / \partial Y = \Phi_1(Z) g_1(Y) > 0$$

Since $g_1(Y) > 0$ and $\Phi_1(Z) > 0$ (equation 6.4.5).

$$(6.4.8) \quad \partial^2 \Phi(Z) / \partial Y^2 = (g_1(Y))^2 \Phi_{11}(Z) + \Phi_1(Z) g_{11}(Y) > 0$$

since $\partial \Phi_1(Z) / \partial Y = \Phi_{11}(Z) g_1(Y)$.

The amount of expenditure decreases as more of the input production is internalized since both the price paid for smaller volumes of input purchases as well as the transaction costs are reduced. Therefore,

$$(6.4.9) \quad \partial \Phi(Z) / \partial I = - \Phi_1(Z) < 0$$

$$(6.4.10) \quad \partial^2 \Phi(Z) / \partial I^2 = - \Phi_{11}(Z)(-1) > 0$$

The cross-partial derivatives of the $\Phi(Z)$ function are,

$$(6.4.11) \quad \partial^2 \Phi(Z) / \partial Y \partial I = g_1(Y) \Phi_{11}(Z)(-1) < 0$$

$$\text{since } \partial^2 \Phi(Z) / \partial Y \partial I = \partial \Phi_1(Z) / \partial Z g_1(Y) \partial Z / \partial I$$

From the properties listed in (C) and 6.4.9, the expected behaviour of $\Phi(Z)$ is depicted in Figure 6.4.0. Since, at all values of $I > g(Y)$ the firm is able to sell the surplus input on the market,⁶ an extra unit of input would be able to fetch $\bar{Z} p(\bar{Z})$ as revenue to the firm. $T(Z)$ becomes zero for all values of $I \geq g(Y)$. Therefore the expenditure function becomes flatter after $I = g(Y)$. It is monotonically decreasing up to $I = g(Y)$. Therefore there is a kink at the point $I = g(Y)$. The corresponding $\Phi_1(Z)$ and $-\Phi_1(Z)$ is depicted in Figure 6.4.1.

This completes the properties of the expenditure function. The next section develops the concept of an efficient level of integration that should be undertaken, given an initial level of input requirement.

6.5 Efficiency of Vertical Integration

A) Efficiency Defined:

Vertical integration has been deemed to be efficient only if total welfare can be increased through a reduction in the end product prices vis-a-vis a non-integrated firm. Recall from Chapters Three and Four, that the total surplus would increase and prices decrease, when the relevant marginal

⁶The other possibility not considered in the present study is that the extra I may be held as inventory.

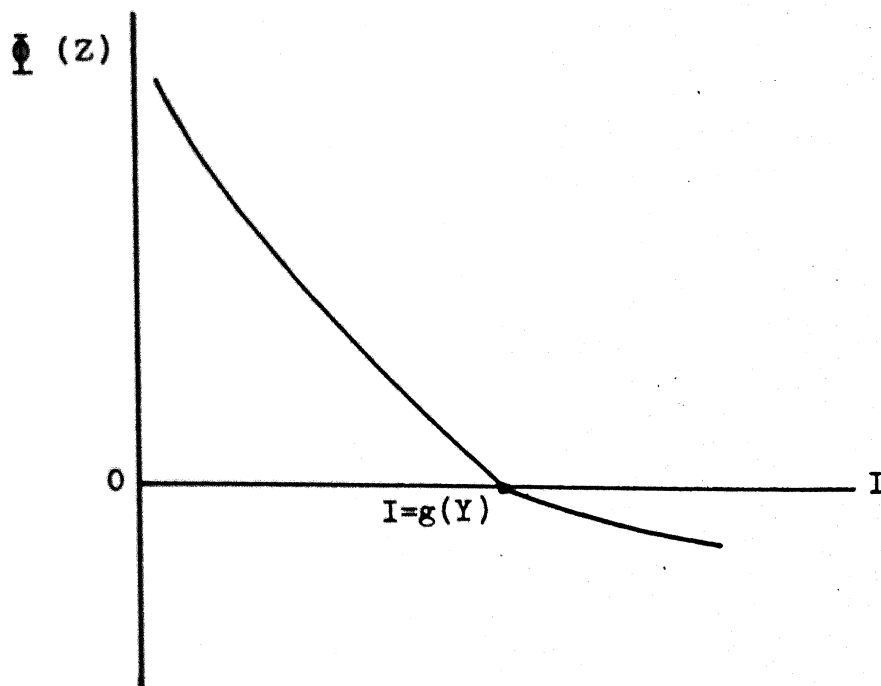


Figure 6.4.0.

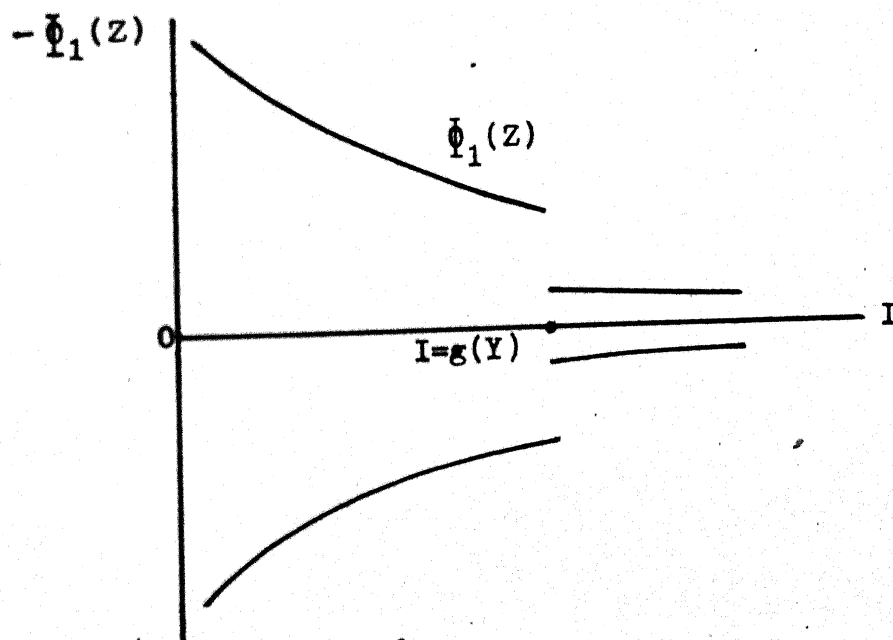


Figure 6.4.1.

costs are shifting downwards given the demand curve. The total variable costs of production were most efficiently allocated between products only when $C_{12}(\cdot) = C_{21}(\cdot) = 0$ is met. Given the demand curves, this condition also ensures maximum total surplus. In the present study it is important to observe that any cost savings due to vertical integration can arise (in the absence of cost complementarity in the joint production of Y and X internally), only due to the savings in the price cost margin and transaction costs. Since both the price of input obtained on the market as well as transaction costs are seen to vary with the level of output Y, these take the form of variable costs of producing a given level of Y. Thus if we ruled out economies of joint production, then the decrease in the costs of producing a given level of Y are only due to a decrease in the variable costs of production of Y. From 6.4.4, it is clear that the total costs come down with integration only if: $\partial C(Y^0, I)/\partial I = C_2(Y^0, I) < 0$ which means that,

$$(6.5.1) \quad \partial \bar{C}(Y^0, I)/\partial I + (-\Phi_1(Z)) < 0$$

The level of integration that minimizes the costs of producing a given level Y^0 , would be governed by the equation,

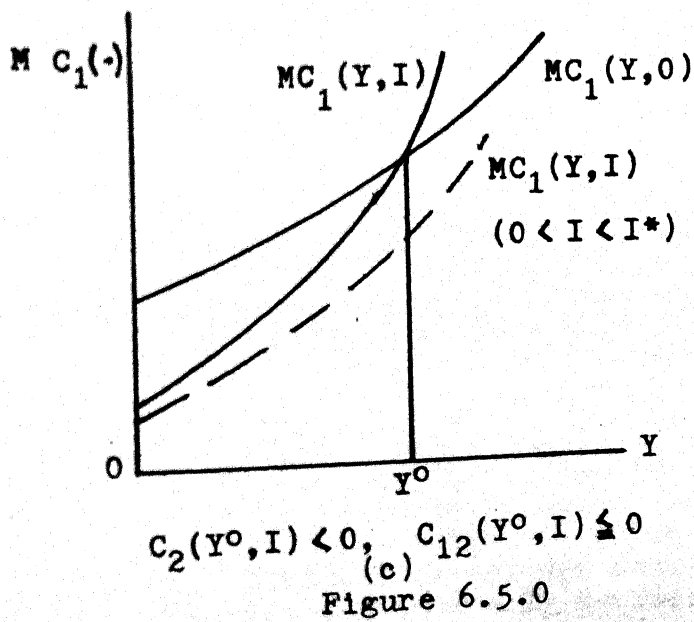
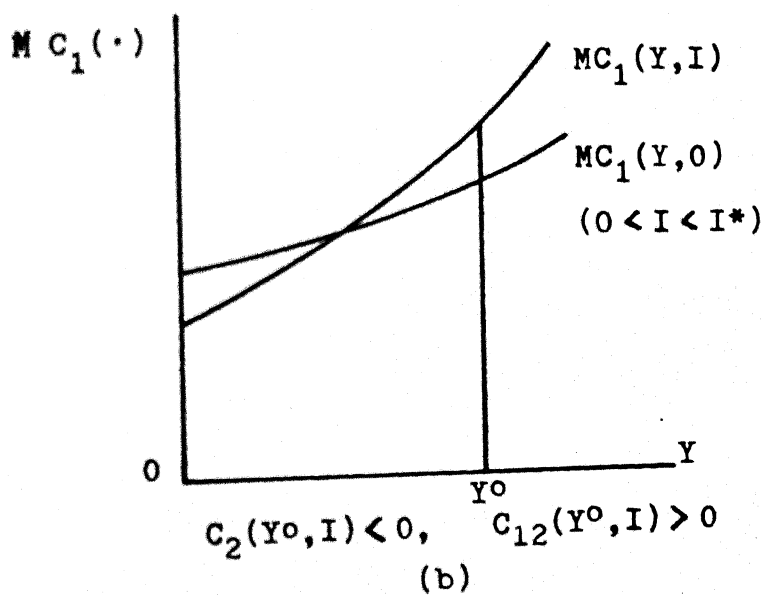
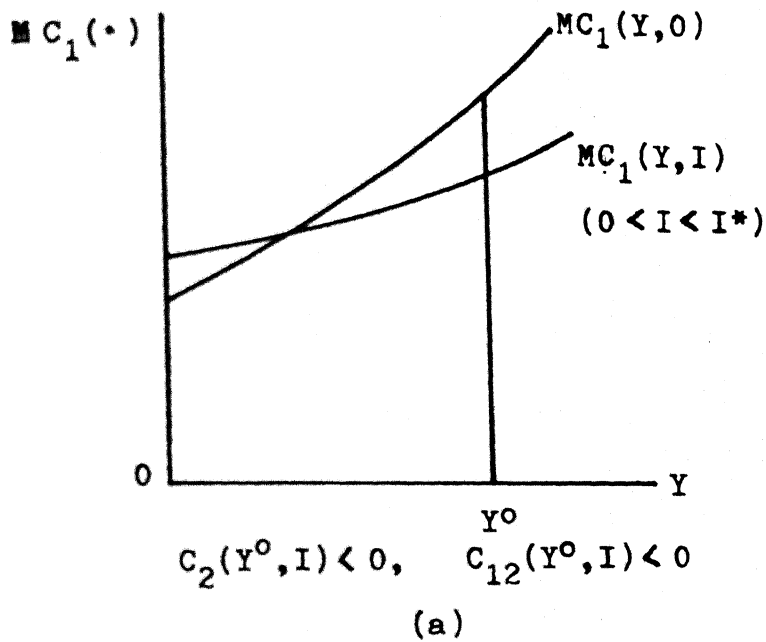
$$(6.5.2) \quad C_2(Y^0, I) = 0 \Rightarrow \bar{C}_2(Y^0, I) = \Phi_1(Z)$$

where, $\bar{C}_2(\cdot)$ represents the marginal costs of producing the input internally whereas $\Phi_1(Z)$ represents the marginal costs of obtaining an extra unit on the market. The firm would find it to its advantage to vertically integrate into the production

of X until the marginal cost of internal production is equal to marginal cost of external procurements. Let the level of integration which satisfies 6.5.2 be I^* . Equation 6.5.1 together with equation 6.5.2 imply that $C(Y^0, I^*) < C(Y^0, 0)$. Therefore,

$$(6.5.3) \quad \int_0^{Y^0} C_1(\theta, 0) d\theta > \int_0^{Y^0} C_1(\theta, I^*) d\theta$$

since the area under the marginal cost curve is equivalent to the total costs of production of Y^0 . However, if economies of joint production are ruled out, then the relative shift in the marginal cost curves that is necessary for 6.5.3 can take any one of the following possibilities depicted in Figure 6.5.0(a,b,c). The third possibility shown in Figure 6.5.0(c) with the dotted MC curve, is that the marginal cost curve is monotonically decreasing. It is evident, therefore, that unless the marginal cost curves shift monotonically downwards in the relevant range, $C_{12}(\cdot) = C_{21}(\cdot) = 0$ will not apply as the cost efficient choice of the levels of Y and I. There are, however, no a priori reasons as to which of these possibilities will hold or not hold; much would depend upon the specific nature of the cost functions. Given the downward sloping demand curve of Y, if the cost savings due to vertical integration reflected by the shifts in the marginal cost curves take the possibilities shown in Figures 6.5.0(a) or (c) then it would result in a fall in end product prices, and part of the gains of the producers are passed on to the consumers.



Notice that if the marginal costs shift monotonically in the relevant range, then,

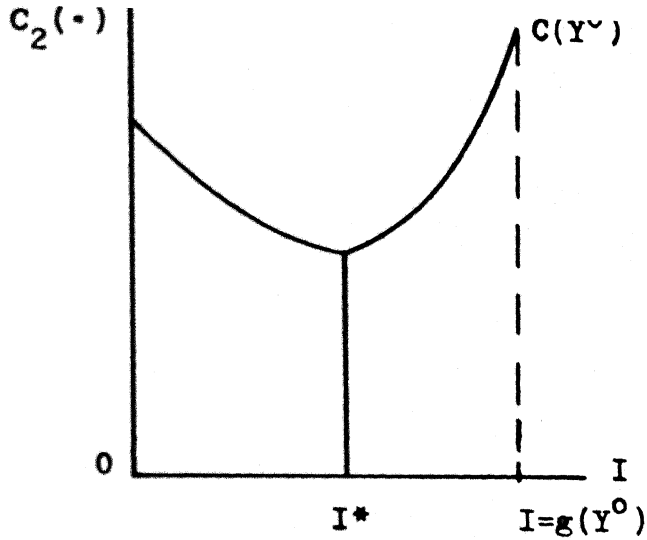
$$(6.5.3) \quad C_2(Y^0, I) \leq 0 \Rightarrow C_{12}(Y^0, I) \leq 0 \quad \text{as } I \leq I^*$$

In the analysis that follows, it is assumed that 6.5.3 holds. Although without economies of joint production this would seem to be a special case, it is perfectly plausible when there are economies of joint production in the production of Y and I. The more general case is illustrated in Section 6.8. Generally, given a level of Y, there would exist a level of X which would be internalized in the interests of the cost advantages available through such internalization. To the extent that transaction costs are negligible, buying, rather than internalizing would normally be the most cost effective means of procurement. The above assumptions and arguments are equivalently depicted in Figure 5.5.1(a,b,c). Notice that the optimal value of I need not correspond to the full integration value $I = g(Y)$. Therefore full integration is neither necessary nor sufficient for efficiency of vertical integration. Further, at $I = g(Y)$, the total costs are far greater than even the non-integrated stage.⁷

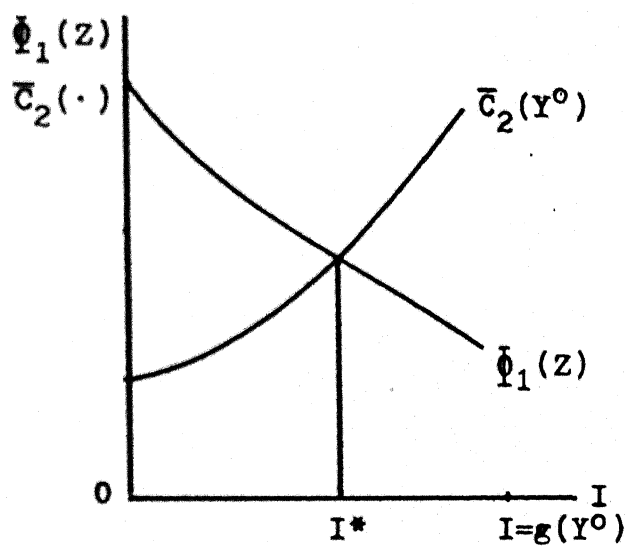
B) Nature of the Y-I* Relationship

The next logical step is to examine the nature of the changes in the efficient value of I^* , when Y changes also.

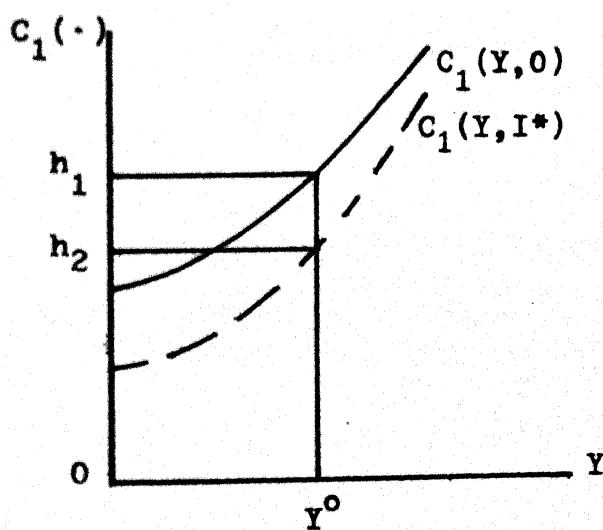
⁷The other interesting possibility is that the total costs of a fully integrated firm are still lower than the non-integrated stage. It can be seen that, even if this holds, the cost minimization behaviour (when there are no economies of joint production) and/or the cost efficient choices (when there are economies of joint production) may not correspond to $I = g(Y)$ unless total costs are monotonically decreasing up to $I = g(Y)$.



(a)



(b)



(c)

At the optimum, equation 6.5.2 holds. Changes in the value of Y will have the effect of changing the $\bar{C}_2(\cdot)$ function as well as the $\Phi_1(Z)$ function. Specifically, from the assumption that there are no economies of joint production,

$$\partial \bar{C}_2(\cdot) / \partial Y > 0$$

$$(6.5.4) \quad -\partial \Phi_1(Z) / \partial Y = -\Phi_{11}(Z) g_1(Y) < 0$$

Since $\Phi_1(Z)$ is a negative quantity, 6.5.4 implies that this is getting larger in numerical magnitude. Thus the relative shift in $\Phi_1(Z)$ in the positive orthant shown in Figure 6.5.2 would be upwards towards the right. Similarly, the $\bar{C}_2(\cdot)$ would move upwards towards the left. The direction of change in the optimal I^* would therefore depend upon the relative magnitudes of the shifts in both these curves. A larger initial Y on the one hand increases the internal costs of production of I which may be a disincentive to vertical integration. On the other hand, the increasing transaction costs of going to the market (since $g_1(Y) > 0$) would be an incentive to integrate. Initially, if the diseconomies in producing Y and I together are small in relation to the marginal increases in transaction costs, the efficient choice of I would bear a complementary relationship with Y . However, it is reasonable to suppose that eventually the increases in the costs of producing jointly may be greater than going to the market. In such a case the efficient I and Y would move in opposite directions. These possibilities have been shown in Figure 6.5.2.

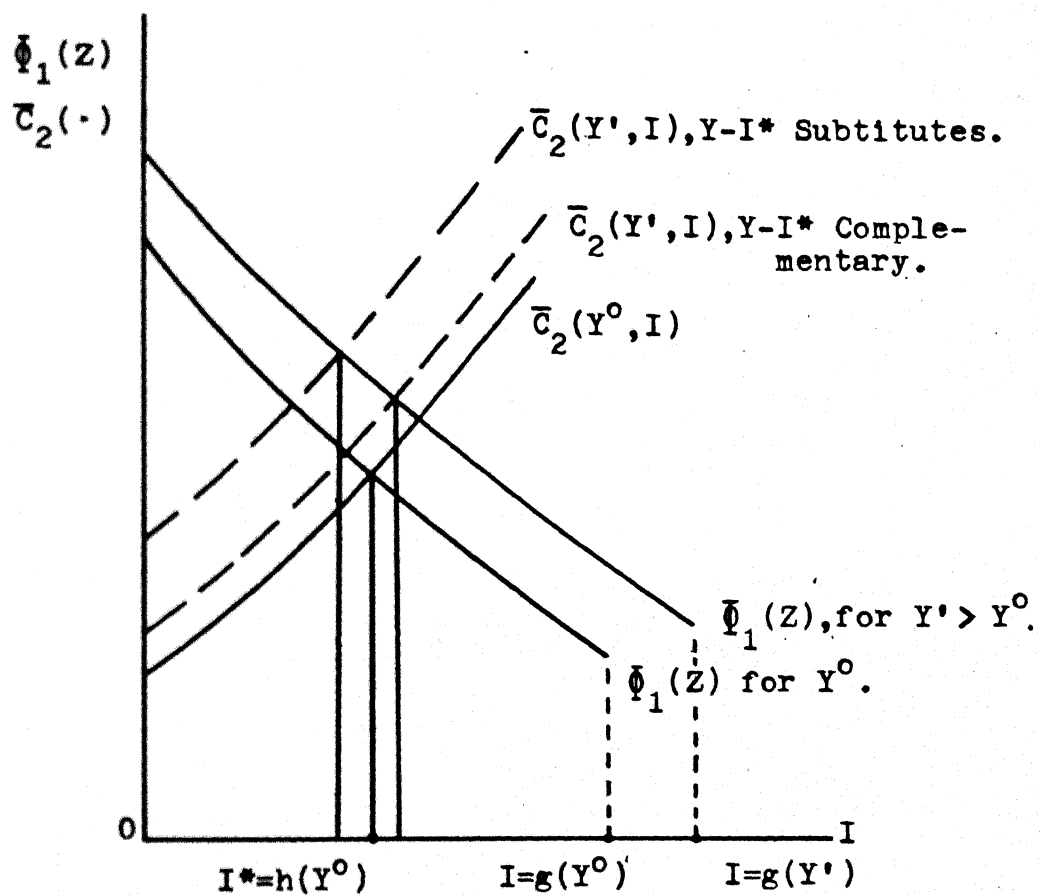


Figure 6.5.2.

If there are complementarity relationships in the initial stages of integration and later on for larger values of Y the firm is better off buying more on the market, the locus of the point at which $C_2(\cdot) = 0$ for changing values of Y , would also constitute the locus of the E-boundary on account of equation 6.5.3. The shape of the boundary is shown in Figure 6.5.3. Similarly, if the economies obtainable by avoiding the market are always greater than the diseconomies of producing jointly, then the locus of points for which $C_{12}(\cdot) = C_{21}(\cdot) = 0$ would be a positively sloped line in the Y - I plane. However, such a possibility seems unlikely. Lastly, if the costs of producing internally become higher than the transaction costs incurred on the market for larger Y , then the E-boundary would have the same characteristics postulated in Chapter Three. The actual shape of the boundary is not as important as the properties of the marginal cost curve for Y when I changes, while operating within the boundary. It is sufficient to note that whatever be the value of Y , given the possibility of saving total costs of production by integration, the marginal cost of Y would shift as indicated in Figure 6.5.0(c), i.e., in a monotonic fashion.

To complete the analysis in this section, the effect of economies of joint production within the firm upon the incentive to integrate must be examined. It is quite obvious that given cost complementarity in the production of Y and I , the incentive to integrate would be that much more enhanced. The definition of the E-boundary, would intuitively

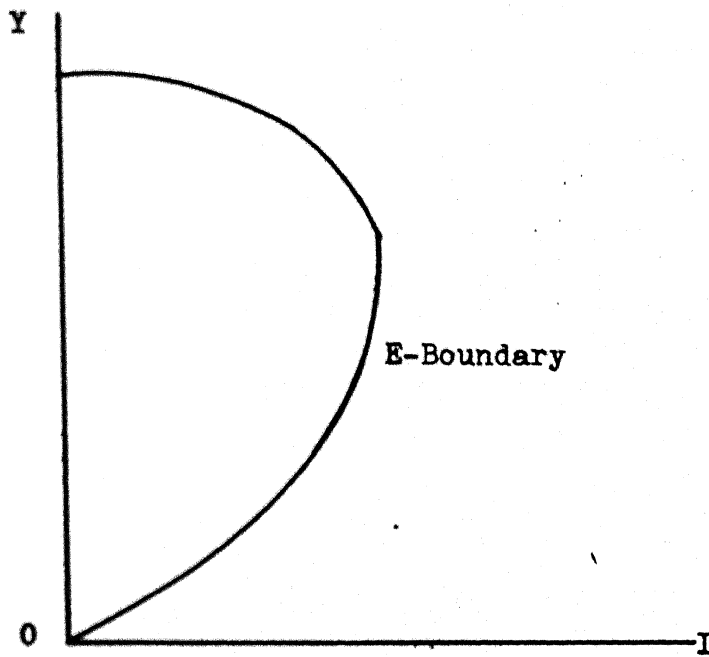


Figure 6.5.3

speaking, account for any economies of joint production if and when they exist.⁸ At a more formal level, let the set of Y-I values that belong to the E-boundary be represented by the set S.

Let $C_{12}(\cdot) \leq 0$ as $I \leq I^*$ for a given Y^0 .

$$\therefore C_{12}(Y^0, I^*) = C_{21}(Y^0, I^*) = 0$$

$\therefore Y^0, I^*$ belong to S.

But, $C_{12}(Y^0, I) = \bar{C}_{12}(Y^0, I) - \Phi_{11}(Z)g_1(Y^0) \leq 0$ as $I \leq I^*$
at $I = I^*$, $\bar{C}_{12}(Y^0, I^*) = \Phi_{11}(Z)g_1(Y^0)$

Internal economies of joint production can be characterized in the manner in which it was done in Chapter Three. For any given Y^0 , $\bar{C}_{12}(Y^0, I) \leq 0$ as $I \leq I^0$. Let the values of Y-I satisfying $\bar{C}_{12}(\cdot) = \bar{C}_{21}(\cdot) = 0$ belong to \tilde{S} . Then, \tilde{S} is a subset of S since, at $I = I^*$,

$$\bar{C}_{12}(\cdot) = \Phi_{11}(Z)g_1(Y^0) > 0$$

Therefore $I^0 < I^*$. Supposing on the other hand, it is assumed that $\bar{C}_{12}(Y^0, I) = 0$ at a value $I^0 > I^*$. Then, at $I = I^*$,

$$\bar{C}_{12}(\cdot) < 0$$

$$\Rightarrow \bar{C}_{12}(\cdot) < \Phi_{11}(Z)g_1(Y^0) \quad \text{since } \Phi_{11}g_1(Y) > 0.$$

$$\Rightarrow \bar{C}_{12}(\cdot) - \Phi_{11}(Z)g_1(Y^0) < 0$$

$$\Rightarrow C_{12}(\cdot) < 0 \quad \text{at } I = I^* \quad \text{which is a contradiction since}$$

⁸This does not mean that the values of Y and I for which the E-boundary was defined above would remain the same when there are economies of joint production. In fact the efficient level of I is shown to be higher than before.

$C_{12}(\cdot) = 0$ for $I = I^*$. Therefore, all values of Y and I belonging to \tilde{S} such that, $\bar{C}_{12}(\cdot) = \bar{C}_{21}(\cdot) = 0$ must belong to S . Therefore, \tilde{S} is a subset of S . It is not necessary that the values of Y and I that belong to S must also belong to \tilde{S} since there may be further incentives to integrate due to transaction costs even when internal economies of joint production are exhausted.

It would be instructive to examine the relative values of efficient choices of I (Y is given) when there are internal economies vis-a-vis not having any such economies to start with.

Since I^* represents the efficient choice of I when $Y = Y^0$, this implies $\bar{C}_{12}(\cdot) = \phi_{11}(Z) g_1(Y)$. When $\bar{C}_{12}(\cdot) > 0$ always is to be contrasted with $\bar{C}_{12}(\cdot) = 0$. Since $\bar{C}_{12}(Y^0) \leq 0$ as $I \leq I^0$, and $I^0 < I^*$ has already been shown. When $\bar{C}_{12}(\cdot) > 0$, then the relative positions of the two possibilities are shown in the following Figure 6.5.4. Let the value of I for which $C_{12}(\cdot) = 0$ when $\bar{C}_{12}(\cdot) > 0$, be \bar{I} . It is clear from the figure that $\bar{I} < I^0$. Therefore economies of joint production would make for higher levels of integration when they exist. The E-boundary is still definable and exists. The result would be valid even if $\phi_{11}(Z) g_1(Y)$ was negatively sloped. This completes the analysis of the efficient levels of Y and I . In the next section, the welfare maximizing as well as the profit maximizing values are derived and shown to correspond to the efficient choices described above given the assumption of monotonicity of marginal costs. Much of the economic

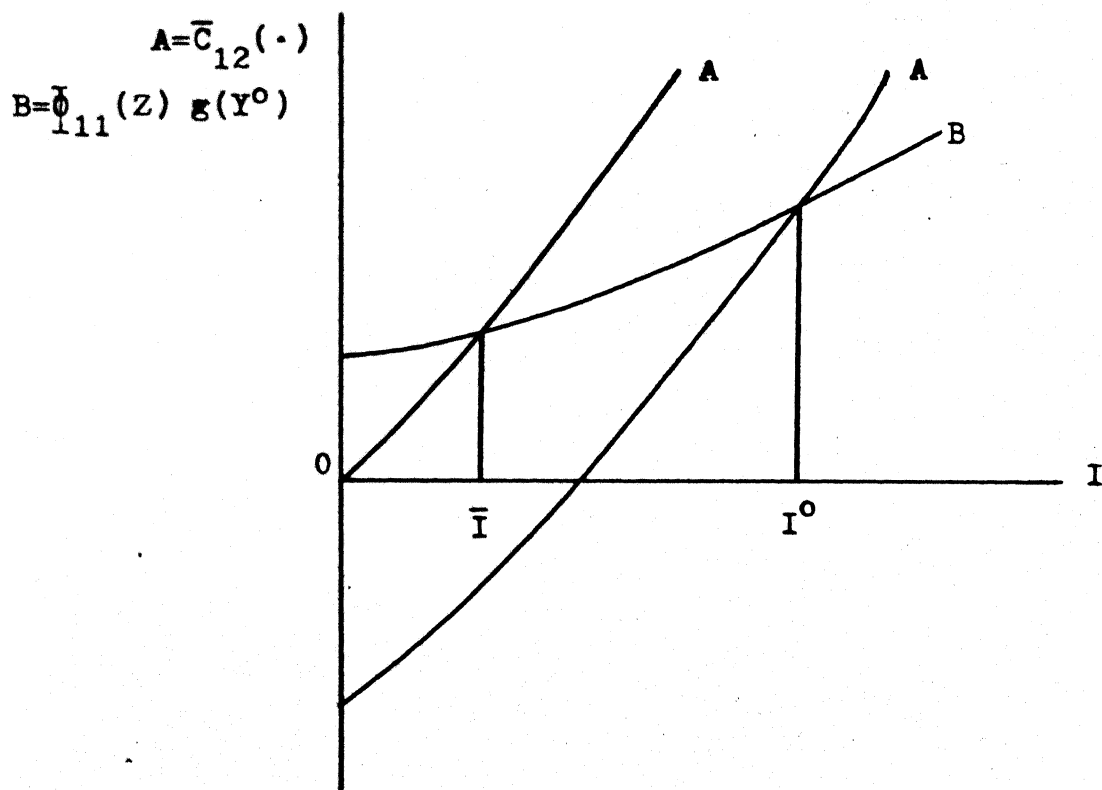


Figure 6.5.4.

reasoning behind the following results parallel to those developed already in the case of horizontal integration with independent demands.

6.6 Welfare, Profits and Efficiency of Vertical Integration: The Case of Strict Cost Complementarity

Let $p = f(Y)$ represent the inverse demand function faced by the firm. We assume that the firm operates in a monopolistically competitive environment and hence it is assumed that $\partial p / \partial Y = f'(Y) < 0$. Let $C(Y, I)$ represent the total cost function whose properties have been discussed in the previous sections; in particular, it is assumed that equation 6.5.3 holds. The total surplus

$$(6.6.1) \quad S = \int_0^Y f(\theta) d\theta - C(Y, I)$$

Maximizing S for values of Y and I yield the first order conditions

$$(6.6.2) \quad S_1 = \partial S / \partial Y = f(Y) - C_1(Y, I) = 0$$

$$(6.6.3) \quad S_2 = \partial S / \partial I = -C_2(Y, I) = 0$$

where $C_1(Y, I) = \bar{C}_1(\cdot) + \Phi_1(Z) g_1(Y)$

$$C_2(Y, I) = \bar{C}_2(\cdot) - \Phi_1(Z)$$

The second order conditions require that,

$$(6.6.4) \quad S_{11} = \partial^2 S / \partial Y^2 = f'(Y) - C_{11}(Y, I) < 0$$

$$(6.6.5) \quad S_{22} = \partial^2 S / \partial I^2 = -C_{22}(Y, I) < 0$$

$$(6.6.6) \quad S_{11} S_{22} - S_{12}^2 > 0 \quad \text{where} \quad S_{12} = \partial^2 S / \partial Y \partial I.$$

Differentiating 6.6.2 and 6.6.3 totally and rearranging the terms yield the trajectory of the Y and I values satisfying the first order conditions:

$$(6.6.7) \quad \left(\frac{dY}{dI}\right)_{E_1} = \frac{C_{12}(Y, I)}{[f'(Y) - C_{11}(\cdot)]} \gtrless 0 \text{ as } C_{12}(\cdot) \gtrless 0$$

$$(6.6.8) \quad \left(\frac{dY}{dI}\right)_{E_2} = -\frac{C_{22}(\cdot)}{C_{21}(\cdot)} \gtrless 0 \text{ as } C_{12}(\cdot) \gtrless 0 ,$$

for $C_{21}(\cdot) = 0$, equation 6.6.8 will take the value ∞ .

Notice that for surplus to be a maximum, equation 6.6.3 is necessary. This, together with the assumption that equation 6.5.3 holds would bring back $C_{12}(\cdot) = 0$ as the solution to the maximisation problem. Otherwise, $C_{12}(\cdot) < 0$ or $C_{12}(\cdot) > 0$ would still be valid solution possibilities. Therefore, we have shown the following proposition:

PROPOSITION 1: The welfare maximizing choice of the level of integration, when transaction costs are significant would always correspond to the efficient choices of the output and input levels, developed in Section 6.5. If, in addition equation 6.5.3 is assumed, then the efficient level of integration would also be 'cost efficient' as defined in Section 3.3, Chapter 3.

It is to be emphasized that the level of integration, denoted by I^e need not correspond to the full integration possibility, i.e., $I = g(Y)$ is neither necessary nor sufficient

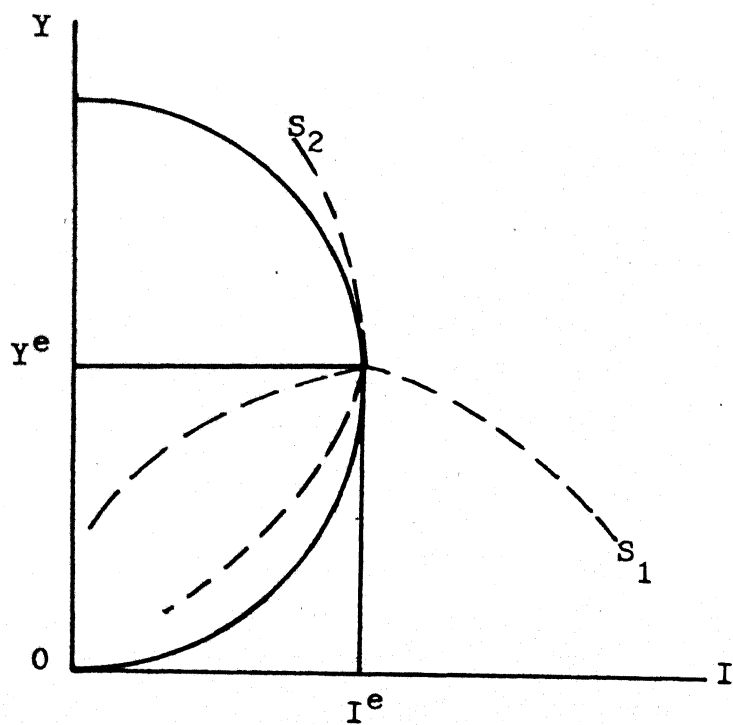


Figure 6.6.0

for maximizing welfare. Equation 6.6.2 corresponds to the familiar Price = Marginal cost rule for a maximum welfare. Equation 6.6.3 amounts to saying that the firm would expand or integrate into the production of X until the costs of organizing an extra unit of input production within the firm is equal to the costs of obtaining that extra unit of input on the market. This is the same condition as 6.5.1 in Section 6.5.

Consider next the profit maximizing choices of Y and I. Let $\pi(Y, I)$ represent the profit function. This can be written as,

$$(6.6.9) \quad \pi(Y, I) = Y f(Y) - C(Y, I)$$

The first order conditions are,

$$(6.6.10) \quad \pi_1(Y, I) = Y f'(Y) + f(Y) - C_1(Y, I) = 0$$

$$(6.6.11) \quad \pi_2(Y, I) = -C_2(Y, I) = 0$$

without going into the algebra, it is sufficient to notice that equation 6.6.11 is the same as equation 6.5.1 which is the efficient level of integration. Therefore,

PROPOSITION 2: The profit maximizing solutions for the level of vertical integration would also be efficient as defined in Section 6.5.

Corollary: Given a size of the firm, full vertical integration is neither necessary nor sufficient for welfare maximization or profit maximization.

The size of the firm is defined to include both the size of the plant as well as the organizational overheads embodied in the firm. The larger the size of the firm, the more efficiently can it conduct transactions on the market, but the larger would be the volume of transactions also. In the next section, changes in the size of the firm is **postulated** and the possibility of a firm being fully vertically integrated examined.

6.7 Changes in Size, Economies of Scope and Full Vertical Integration

Changing size of the firm would also mean changing the scale of operations and hence any cost advantages would be due to economies of scale. Let F be representative of all the other inputs, both organizational and technological, that were held constant throughout the preceding analysis, plus the fixed factors. Redefine the cost function as $C(Y, I, F) = \bar{C}(Y, I, F) + \Phi(Z(F))$. Changing the scale of operations would mean,

- (a) Increasing the capacity of producing both Y and I , and
- (b) A more efficient network in dealing with market transactions.

However, notice that increasing the capacity of Y and hence $g(Y)$ would entail a larger volume of transactions and more complex contractual arrangements with input suppliers.

Therefore the three possibilities open are,

- (i) Transaction costs increase with an increase in the scale of operations.

- (ii) Transaction costs remain same with an increase in the scale of operations.
- (iii) Transaction costs decrease with an increase in the scale of operations.

If there are economies of scale in operation, these would tend to shift the $\bar{C}_2(\cdot)$ curve downwards towards the right since the returns to total-outlays are greater.

$\bar{C}_{23}(\cdot) = \partial \bar{C}_2(\cdot) / \partial F < 0$ up to some value of F which would be optimum. When the size of firm is varied,

$$(6.7.1) \quad Y = \delta(F), \quad \partial Y / \partial F = \delta'(F) > 0$$

$$(6.7.2) \quad I = \theta(F), \quad \partial I / \partial F = \theta'(F) > 0$$

and since $Z = g(Y) - I$,

$$(6.7.3) \quad Z = Z(F) = [g\{\delta(F)\} - \theta(F)]$$

$$\frac{\partial Z}{\partial F} = g'(Y) \cdot \delta'(F) - \theta'(F)$$

$$(6.7.4) \quad \Phi(Z) = \Phi(Z(F))$$

$$\begin{aligned} (6.7.5) \quad \partial \Phi(Z) / \partial F &= Z \frac{\partial p(Z)}{\partial F} + p(Z) \frac{\partial Z}{\partial F} + T'(Z) \frac{\partial T(Z(F))}{\partial F} \\ &= Z p'(Z) [g'(Y) \delta'(F) - \theta'(F)] + p(Z) [g'(Y) \delta'(F) \\ &\quad - \theta'(F)] + T'(Z) [g'(Y) \delta'(F) - \theta'(F)] \\ &= [g'(Y) \delta'(F) - \theta'(F)] [Z p'(Z) + p(Z) + T'(Z)] \end{aligned}$$

if $g'(Y) \delta'(F) = \theta'(F)$ then, $\partial \Phi(Z) / \partial F = 0$ and the expenditure function remains invariant to changes in size. However, since $\bar{C}_{23}(\cdot) < 0$, full integration would take place if and only if the economies with respect to size are so as to make

the cost minimizing choice of I equal to $g(Y)$. This is shown in Figure 6.7.0. However, if 6.7.5 is positive, then the relative economies of size must be large enough to overtake any increase in the transaction costs to bring about full integration.

Economies of Scope and Full Integration

Recall from Section 6.5 that the existence of internal economies of scope in the production of Y and I are an added incentive to integrate. This means that, as Y and I are increased in the relevant range $\bar{C}_{12}(\cdot) = \bar{C}_{21}(\cdot) < 0$ and hence the $\bar{C}_2(\cdot)$ curve shifts downwards to the right. At the same time larger values of Y would mean a higher value of $g(Y)$ and higher the transaction costs. Again, therefore, full integration will be efficient if the economies of scope are strong enough. This is shown in Figure 6.7.1.

6.8 Further Observations and Conclusions

It was observed that the efficient level of integration was one which minimized the costs of production of a given level of Y . If the assumptions of strict cost complementarity occurring *pari passu* with the decrease in total costs due to integration is relaxed, then the E-boundary will not be the guiding factor for efficiency since the optimum is decided by $C_2(\cdot) = 0$ and $C_{12}(\cdot)$ may be negative, zero or positive at that value. Strict economies of joint production over a certain range will bring back the point of exhaustion of cost complementarity as the efficient point. The actual welfare maximizing choices of I and Y do not tell us anything

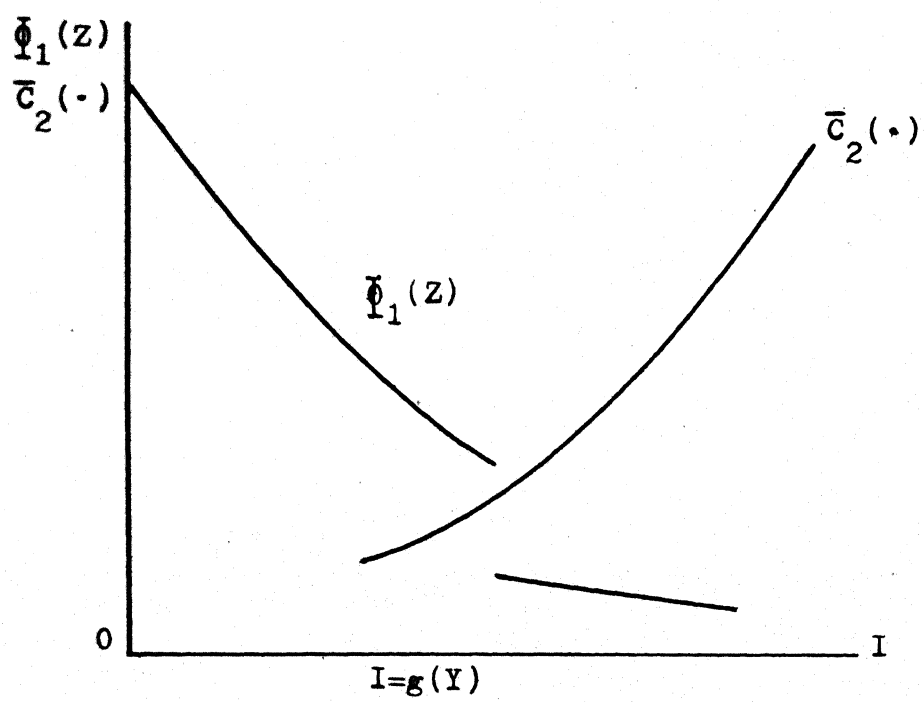


Figure 6.7.0.

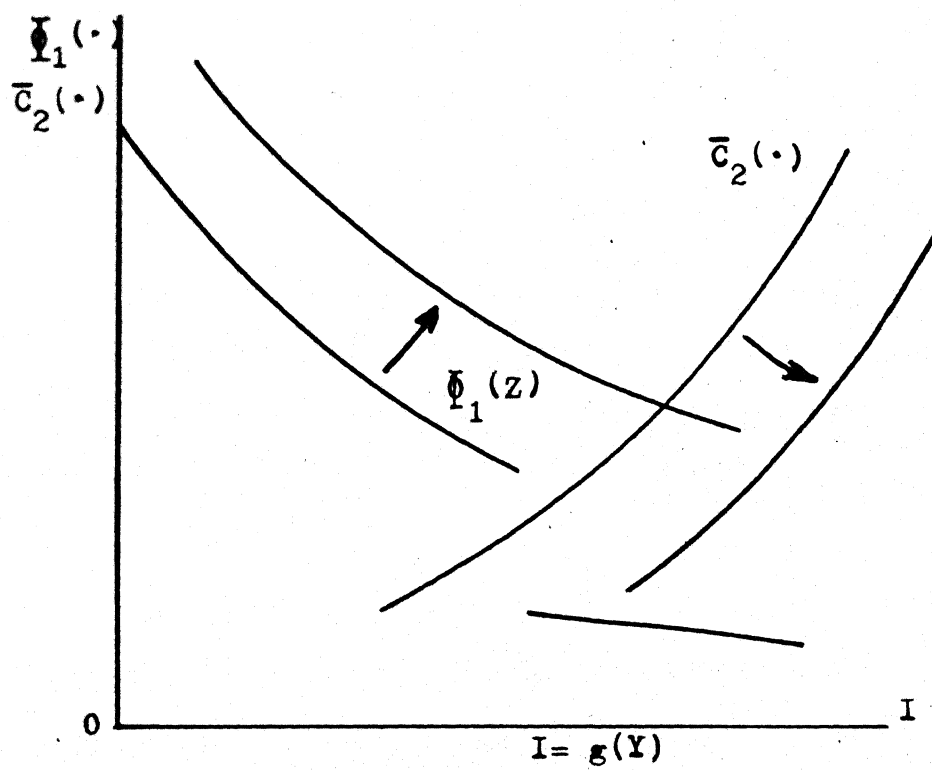


Figure 6.7.1

about the level of integration being partial or full. Again, much would depend upon the specific cost structures postulated. However, it is clear that a fully vertically integrated firm is not necessarily the only outcome for efficiency.

To conclude, it is observed that transaction costs play a crucial role in motivating a firm to undertake vertical integration. For a special class of cost functions, vertical integration can bring about a reduction in end product prices. In this analysis, economies of joint production occurring simultaneously with the incentive to save transaction costs would generally bring about a reduction in end product prices. The profit maximizing degree of vertical integration as well as the surplus maximizing degree of integration, would, under this particular case, always be so as to operate with the minimum marginal cost of producing the given level of Y . In the more general case, the guiding factor is the concept of cost minimizing value of the degree of integration. The end product prices may rise, fall or remain unchanged, and there are no a priori reasons as to what would be the most likely occurrence.

CHAPTER 7

SUMMARY AND CONCLUDING REMARKS

7.1 The Perspective

Firms in the world today are seen to be both technologically and organizationally complex entities offering a variety of products on the market. Some of these are related and others not related to each other commercially. All firms can be conveniently classified as being horizontally integrated, conglomerately integrated or vertically integrated, or a combination of these.

A perusal of the literature shows that the traditional theory of the firm is, in fact, a conglomeration of many theories, each of which explains some particular aspect/s of the firm in real world situations, usually to the exclusion of other aspects. For instance, the technical relationships between inputs and output have been examined by assuming an almost non-existent organizational structure, thus giving rise to the single ownership cum manager concept. Similarly, the effects of market structure upon the pricing decisions of firms have been examined by oversimplifying the multiproduct nature of the firm to one of a single product enterprise.

More recently, the literature offers a deeper insight into the nature of the firm. This has been achieved along three major branches:

- (i) The recognition that there are significant costs in the use of the market and price mechanism.
- (ii) The multiproduct nature of firms.
- (iii) The observed diversity of products that are offered and the impact of market structure on product diversity.

The existence of significant transaction costs makes for the existence of firms in the first place and also provides an explanation of why firms tend to become multiproduct firms by vertically integrating their activities. Similarly, the recent growth of a considerable amount of literature on the multiproduct firm points out that the existence of both organizational as well as technological factors contribute to what is generally referred to as economies of scope and economies of joint production, and that these may create powerful incentives for multiproduct production, i.e., horizontal integration. Product diversity, on the other hand, has been characterized in many studies to the exclusion of multiproduct aspects. The scant evidence of the characterization of product diversity provided by a single firm has ignored economies of joint production and economies of scope.

The overriding question that all these studies addressed themselves to is that of deriving the efficient choices of quantities and prices. Thus, it is in the interests of efficiency that firms (i) avoid transaction costs by integrating vertically, or (ii) exploit economies of scope by integrating horizontally, or (iii) in extreme cases become a natural monopoly.

7.2 The Intended Contribution of the Present Study

Given the broad general perspective that emerges from the above perusal of the literature, several pertinent questions emerge. If economies of joint production is one way multiproduct production may emerge, why should horizontal expansion be treated as a form of merger and not arising from internal expansion which the firm undertakes by exploiting such economies? Similarly why should transaction costs be ignored in studying vertical integration? and why is that the literature on product diversity offered by a single firm does not take into account any economies of scope?¹

The present study was undertaken as an attempt to fill in this gap which is apparent as far as firm level characterizations go. Throughout this study, internal expansion, as being one of the ways, in which firms diversify has been emphasized. It is recognized that the insights developed by Coase, and later Williamson along with other arguments due to Penrose provide persuasive reasons as to how the organizational dimension can contribute to economies of scope. This, along with technological economies can cause diversification through internal expansion. Thus, the concept of economies of joint production was used to describe horizontal expansion whereas the transaction cost arguments were used to portray vertical integration.

¹Ireland (1983) would be the lone exception.

7.3 Summary of the Concepts Used and Methodology

Economies of joint production was characterized using the concept of weak cost complementarity in the production of a product range. The extent of cost complementarity was argued to be limited by the availability of excess capacity in the use of common inputs. Cost complementarity bears a positive relation with excess capacity. The greater is the quantity of the initial product that the firm produces, the less would be the extent of cost complementarity for the production of other products. It is recognized that cost complementarities need not exist indefinitely, and would be limited at any given point of time. To the extent that they are available, the firm would find it to its advantage to exploit these economies to their fullest extent if the market so permits it. Thus the a priori notion of cost efficient choices of output quantities as those that would exhaust the economies of joint production emerged. This was defined in terms of the E-boundary in the models. In the case of vertical integration, the transaction cost arguments, when incorporated explicitly into the cost function have shown that the cost advantages realized by vertical integration are not unlike economies of joint production in the case of horizontal expansion.

On the demand side, four possible relations that can occur between products were identified:

- (i) The products are unrelated.
- (ii) The products are substitutes.

(iii) The products are complements.

(iv) The products have an input-output relation.

The analysis was made a little more general by including variants of the same product produced by the firm under the second group. The interdependencies between products was portrayed through the shifts in the demand curves for changes in the quantities of other products that the firm produces. The first three groups were akin to horizontal expansion whereas the last type of product relationship is, of course, vertical integration.

Using quantities rather than prices, the economically efficient choices of products as well as the profit maximizing choices were derived separately for each type of product relatedness envisaged above, and examined against the background of cost efficient choices of output levels developed in the model. Changes in the efficient quantities of the products were examined when the competitive level (given exogenously) changed. This was done for group one and three.

To extend the concepts of economic efficiency when there are interdependent demands, it was necessary to assume integrability conditions on the demand structures. Further, to keep the analysis to the lowest degree of complexity the number of products that the firm produced was restricted to two. In the vertical integration case, a single, final product was assumed.

7.4 Summary of the Results

(A) Unrelated Products:

The welfare maximizing choices of output quantities, as well as the profit maximizing output levels were shown to correspond to the cost efficient choices only under specific assumption on demand structures. This has several implications:

(i) The E-boundary represents the points of exhaustion of cost complementarities. This may or may not correspond to the point of exhaustion of economies of scope. However, such cost efficient choices clearly generate a larger total surplus/profit than when independent production of the products takes place.

(ii) The net results after integration are higher output quantities and lower prices, thus showing that whenever such economies of joint production exist, the resulting savings in costs may be passed on to the consumers.

(iii) The difference between the profit maximum choices of output levels and the corresponding welfare maximizing levels arises from pricing the products above the marginal costs in the former case. Both the quantities of the products were lower than the surplus maximizing values.

(iv) Both surplus and profits are shown to be monotonic functions of output levels when demand curves are independent.

The benchmark of efficiency was taken to be those output quantities that maximize surplus as well as lead to cost efficient solutions. For this purpose the demand curves were

confined to a particular class which allowed welfare maximising choices of unrelated goods to be cost efficient also.

(B) Substitute Products:

Assuming this class of demand functions, the introduction of substitutability effects were so as to leave output at levels wherein the marginal cost of each is higher than the minimum possible under the welfare and profit maximisation objectives.

(i) The result that the production of substitute products is not cost efficient, is generated solely through the demand interrelationships. When such a interrelationships are removed from the analysis, the bench mark solution is restored. The implication of this is that although the market permits the full exploitation of economies of joint production, the substitution effects of the demand curves preclude the firm from this full utilization of excess capacity in the common inputs. When the firm is envisaging horizontal expansion into product variety, the non-exhaustion of economies of joint production as of economies of scope may initiate the firm to further diversify into other product varieties which have negligible near-neighbour effects.

(ii) In the final outcome it was seen that the relative changes in the equilibrium quantities were most likely to cause a decrease in surplus vis-a-vis the independent demands case.

(iii) The resulting inefficiencies caused by substitution relationships in demand may well cause the firm to minimize the demand interdependence effects by resorting to market

segmentation or selling products to different consumer groups whose valuations of the products are independent of one another.

(iv) Regarding the profit maximizing quantities it may be noted that profits are monotonic functions of equilibrium values of output quantities. Therefore, substitution effects on the demand side may actually reduce overall profits.

(C) Complementary Products:

When the product line was composed of products which were complementary to each other, cost complementarities were reinforced with higher willingness to pay prices due to the outward shifting of the demand curves. Therefore, the equilibrium solutions for total surplus maximization occurs well beyond the region where cost complementarities were shown to be exhausted. And since welfare was a monotonic function of output quantities the most likely outcome was that total surplus may actually go up. The role of the E-boundary in this particular case was passive for the inefficiencies generated by operating outside the boundary were more than offset by increases in the consumers' willingness to pay. In a similar fashion, it was shown that the profit maximizing values also undergo changes in a manner analogous to the surplus maximizing choices.

Effects of Outside Competition:

(A) Unrelated Products:

The effects of changes in the external competitive level for at least one of the firm's products are complicated and

curves and changes in the elasticity of demand for the firm's product. The results of the study show that,

(a) The flatter the demand curve, the more significant would be the deviations from the original surplus maximizing values stated under proposition one.

(b) There would be inefficiency generated, with total surplus reducing whenever competition tends to take away consumers with lower valuations of the firm's product. Due to cost interrelationships, the other products of the firm would also bear a part of these effects.

(c) The profit maximizing quantities are sensitive to both the changes in the slope as well as the elasticity of demand for the product in question. When the demand curves become more elastic due to outside competition the resulting equilibrium configuration was shown to be cost inefficient. However, it was shown that gains in profits were possible in one market parri passu with the firms incurring higher marginal costs in the other product with a decrease in the profit level. Thus, the firm may accept this loss if it finds that it can cross-subsidize the losses from the gains in the other product.

The possibility of the firm dropping a product from its product line has been shown to occur even when elasticity of demand changes favourably. Similarly the effects of competition will result in a deviation from the profit maximizing equilibrium levels whenever the demand curves become less elastic and the infra-marginal revenues obtainable from the

product in question are adversely affected. This result is analogous to the findings of the literature on product diversity. It has been understood from the studies by Spence that products with low elasticities of demand have difficulty surviving on the market. The additional qualifications within the multiproduct setup show that this is true if the infra-marginal revenues are adversely affected.

(B) Complementary Products:

The results in this case turned out to have a large number of possibilities with no a priori reasons as to which of the possibilities are likely to occur. Therefore, the findings were not given the status of a proposition that could actually be proved.

Vertical Integration: Summary of the Results:

The efficient level of integration was observed to be one which minimized the costs of producing a given level of the final product. Transaction costs and the monopolistic nature of the input supplier were shown to play a crucial role in the incentive to integrate even under the absence of internal economies of joint production. The concept of cost efficient output levels of both the final product and input produced internally were identical to that developed for horizontal expansion only if the savings in transaction costs and the monopolistic margin, of input bought on the market, was so as to bring about a monotonic shift in the marginal cost curve of the final product. In such cases, vertical integration would also bring about a reduction in

end-product prices. In the more general case, the concept of cost minimizing value of the degree of integration was shown to be the guiding factor. The welfare maximizing as well as the profit maximizing choices of the degree of integration were shown to be consistent with cost minimizing behaviour. Full vertical integration was neither necessary nor sufficient for cost minimization. The role of economies of scale and scope were analyzed to show that full integration could occur as one of the possibilities.

7.5 Concluding Remarks and Limitations of the Study

The new insights obtained by this study were that, the role played by economies of joint production give a guideline, in terms of the E-boundary, to the economically efficient values under independent demands, since, given the demand curve, the total surplus would increase whenever the marginal costs shift downwards. The construction of the E-boundary makes it possible to get to the actual process by which the various solutions emerge thus giving a much clearer picture of the complex interactions between demand structures and cost structures. The comparison of the total surplus generated under different product groups rests crucially upon the properties of the E-boundary, which enables the comparison of the marginal costs of production at different output quantities. The fact that economies of joint production exist and are finite and limited is itself an indicator of how efficiently the common inputs in the production of a product line are allocated. It was observed that the

sufficiently general functional forms adopted in this study provided a way to exhibit some of the results already existing in the literature. At the same time the multiproduct nature of the firm showed that still other results in the literature may have to be qualified.

At a more general level this study is an attempt to integrate three areas that are studied in the literature, albeit separately; (1) The multiproduct firm, (2) Transaction costs and their effects, and (3) Product diversity; and seek to analyze the welfare and profit maximizing behaviour of such firms.

As with any study, there are limitations in the present case as well.

(i) The analysis has been confined to the two variable case and hence it was not possible to analyze a product mix involving a combination of product groups.

(ii) The case for internal expansion was made through the concepts of economies of joint production, which presupposes the existence of common inputs that, once procured, can be costlessly used for the production of other goods. While this is one way in which internal expansion may occur, it is certainly not the only way. Indeed it must be recognized that the existence of economies of scope need not have cost complementarity in production. Other conditions under which economies of scope occur have not been studied.

(iii) Consumer heterogeneity has been a continued topic of interest in the literature on product diversity;

and may account for demand interrelationships to be less significant than portrayed in this study. To a limited extent this was acknowledged in the discussions on horizontal integration.

(iv) The analysis of a change in external competitive levels is also limited in extent due to the algebraic complexity and diseconomies of using this framework as it is for larger market models.

(v) The model of vertical integration is developed for the particular instance of backward integration only. Input substitution possibility is ignored in the analysis. The reaction of rival firms and the general impact upon the market of such integration has not been examined.

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APPENDIX A

Welfare Maximization Under Alternative Assumptions About Costs and Demand: Some Conjectures

Proposition one (Chapter Four) has established that there exists a class of demand functions for Y_1 and Y_2 (unrelated in demand) for which the welfare maximum solutions satisfy $C_{12}(.) = 0$. Propositions two to six are proved assuming that the initial conditions on demand are such as to lead to $C_{12}(.) = 0$. However, the alternative welfare maximum solutions that would emerge under more general demand conditions may satisfy $C_{12}(.) < 0$ or $C_{12}(.) > 0$. The following analysis heuristically demonstrates propositions two and three under the alternative assumptions. It is conjectured that the analysis would be equally valid if the demand curves are redefined as marginal revenue curves. Hence the analysis can be applied to the remaining propositions under the alternatives $C_{12}(.) < 0$ and $C_{12}(.) > 0$.

Case I : Y_1 and Y_2 Substitutes

(a) To begin with, let the demand curves be such that the welfare maximizing choices of Y_1 and Y_2 for the case of unrelated demands satisfy the condition $C_{12}(.) < 0$.

Let the solution be (\hat{Y}_1, \hat{Y}_2) , and $C_{12}(\hat{Y}_1, \hat{Y}_2) < 0$. Let Y_1 and Y_2 be substitutable instead. Referring to Figures A1 and A2, substitutability effects shift the demand curves D_1 and D_2 to D_1^- and D_2^- . Therefore \hat{Y}_2 reduces to \tilde{Y}_2 . This entails a shift upwards of the marginal cost curve for Y_1 . Together with

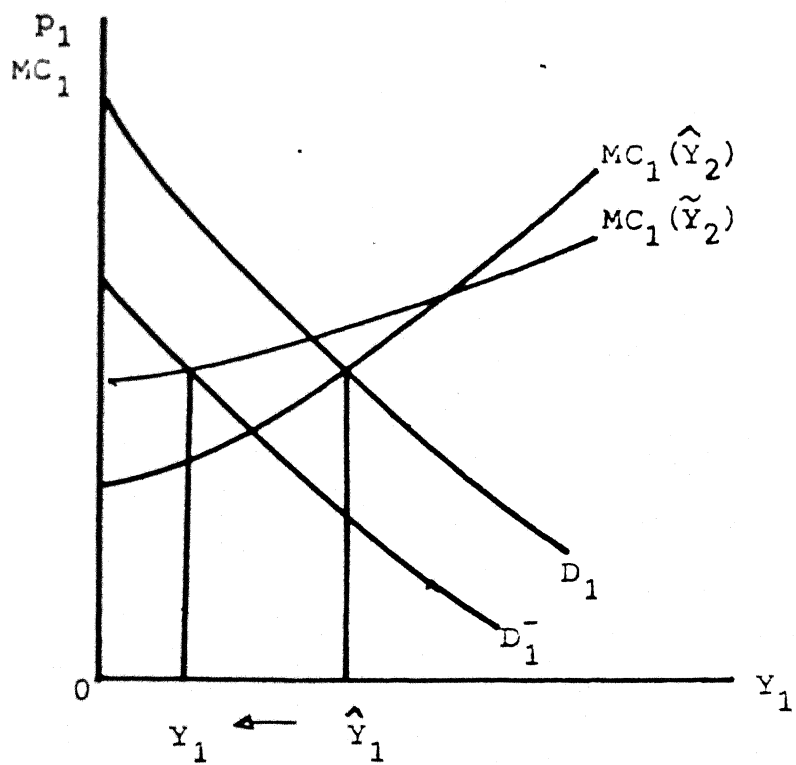


Figure A1

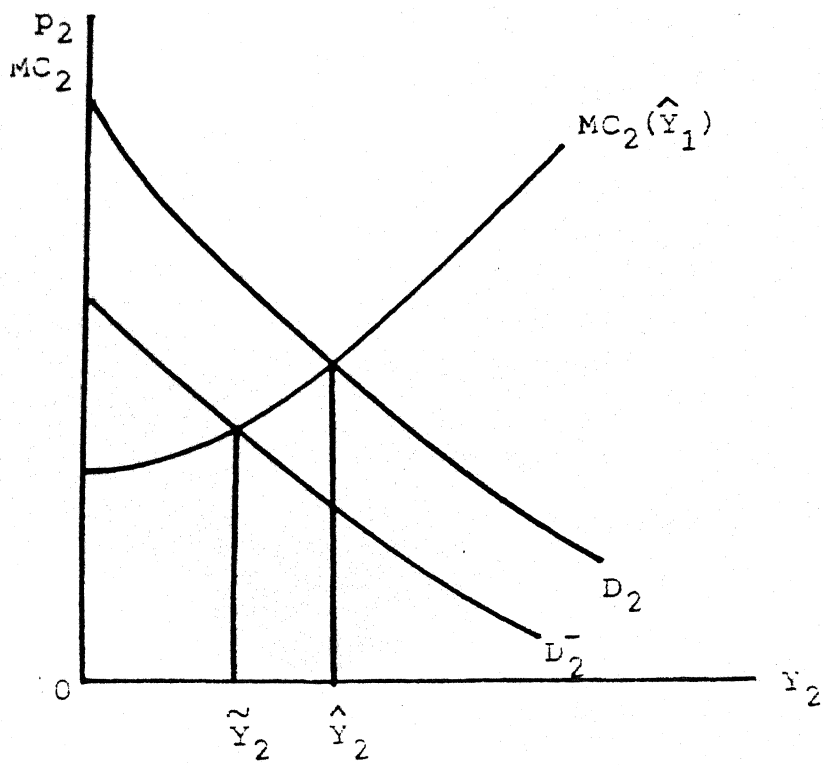


Figure A2

this, a shift in D_1 to D_1^- reduces the value of Y_1 also. The final result is portrayed in Figures A3 and A4. It is obvious that $Y_2^- < \hat{Y}_2$ and $Y_1^- < \hat{Y}_1$. Hence inefficiencies are further compounded.

A similar result holds when the demand conditions are such that the welfare maximum occurs at a point where $C_{12}(.) > 0$:

(b) Let the welfare maximising solution under unrelated demands be represented by Y_1^* and Y_2^* such that $C_{12}(Y_1^*, Y_2^*) > 0$. To simplify the analysis let (Y_1^O, Y_2^O) represent a point on the E-boundary, such that $C_{12}(Y_1^O, Y_2^O) = 0$, and $Y_1^* > Y_1^O$; $Y_2^* > Y_2^O$. Figures A5 and A6 shows the (Y_1^*, Y_2^*) equilibrium in comparison to the (Y_1^O, Y_2^O) configuration and the relative positions of the marginal-cost curves. Such a comparison enables us to investigate the possibility of a cost-efficient solution occurring under substitutability conditions. Consider the intersection of $MC_1(Y_2^O)$ and $MC_1(Y_2^*)$. Such a point F would be to the left of point E in Figure A5, since there exists a value of $Y_1 = Y_1^- < Y_1^O$ for which $MC_1(Y_2^O) = MC_1(Y_2^*)$.

Introducing demand substitutability effects into the analysis reduces the value of Y_1 and Y_2 . However, a reduction in Y_2^* would mean a lowering of the $MC_1(Y_2^*)$. Consider Figure A7. A lower value of Y_2 could mean that it lies between Y_2^* and Y_2^O i.e. Y_2^- such that $Y_2^O < Y_2^- < Y_2^*$. It is then observed that comparing $MC_1(Y_2^*)$ and $MC_1(Y_2^-)$, the value of Y_1 for which $MC_1(Y_2^*) = MC_1(Y_2^-)$ would lie to the left of Y_1^O . This, in turn may be at point F, or not, but is definitely to the left of E. Then, it is clear that the final equilibrium values of Y_1^- and Y_2^- would be both

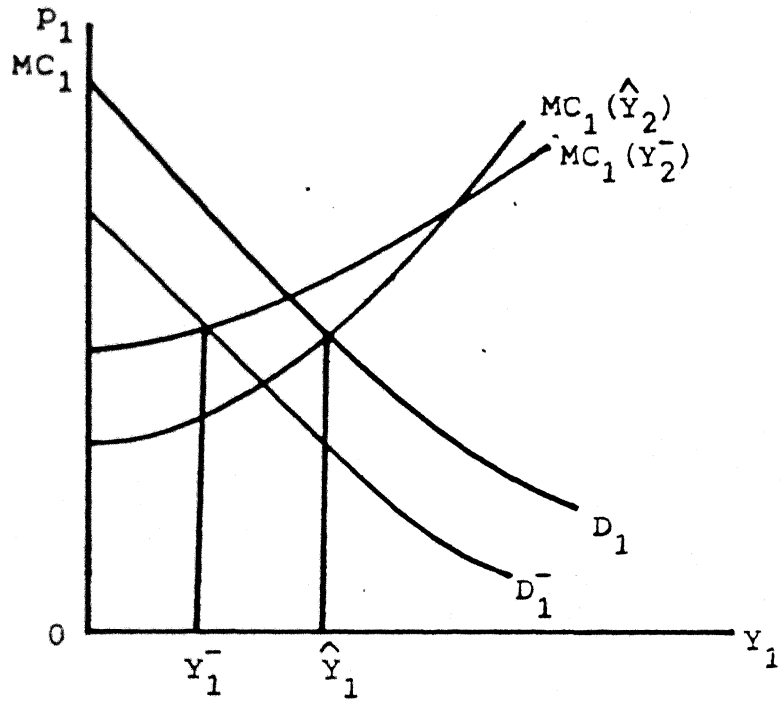


Figure A3

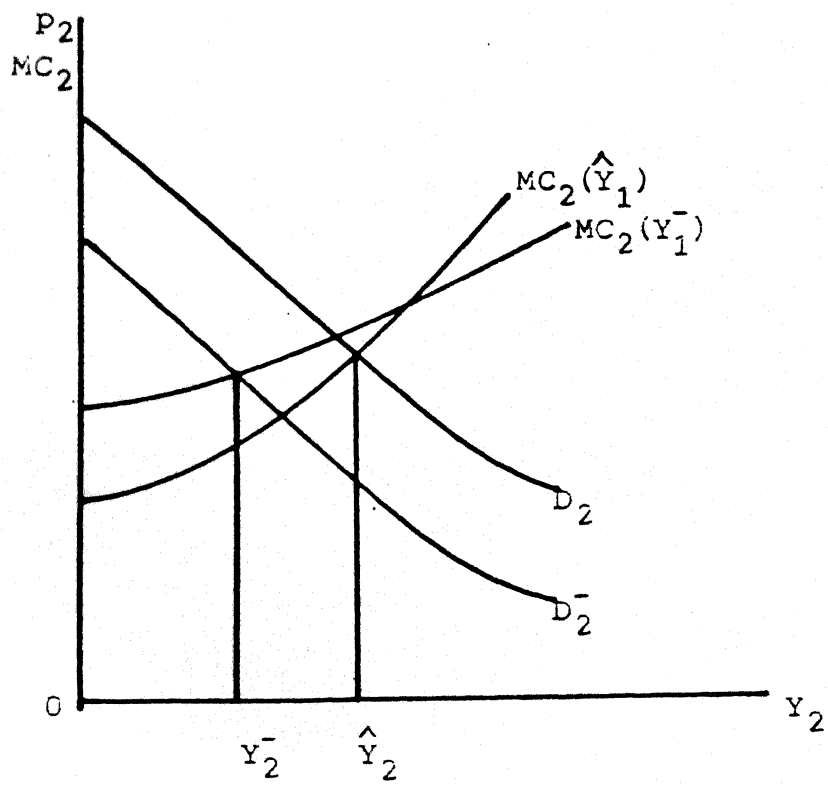


Figure A4

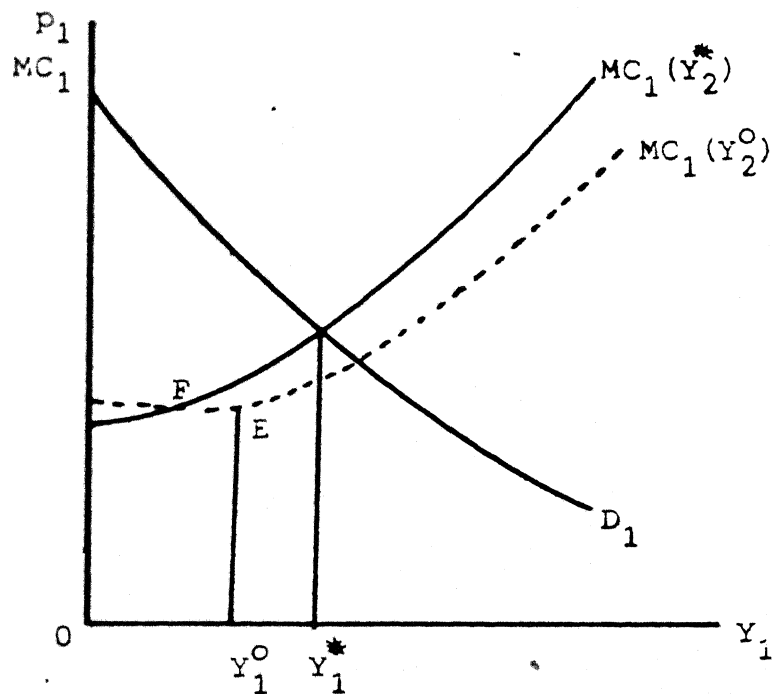


Figure A5

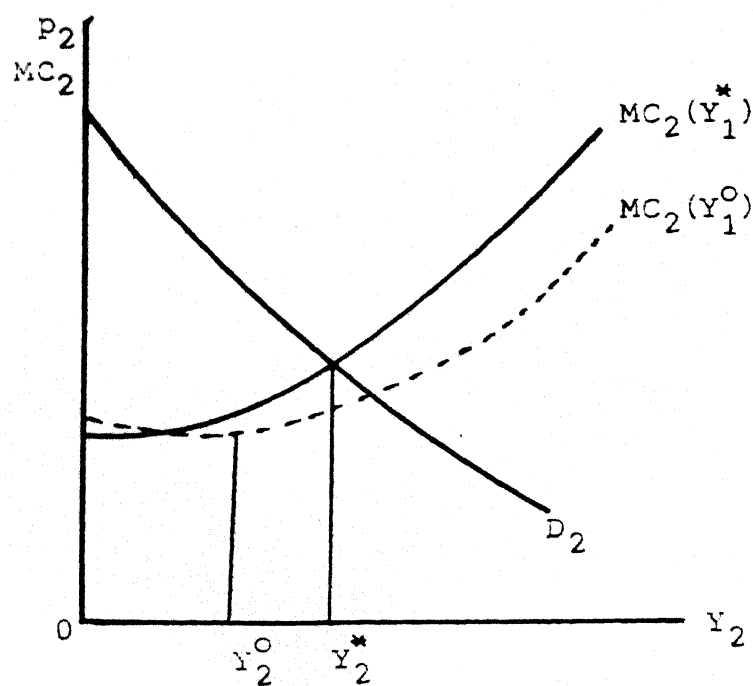


Figure A6

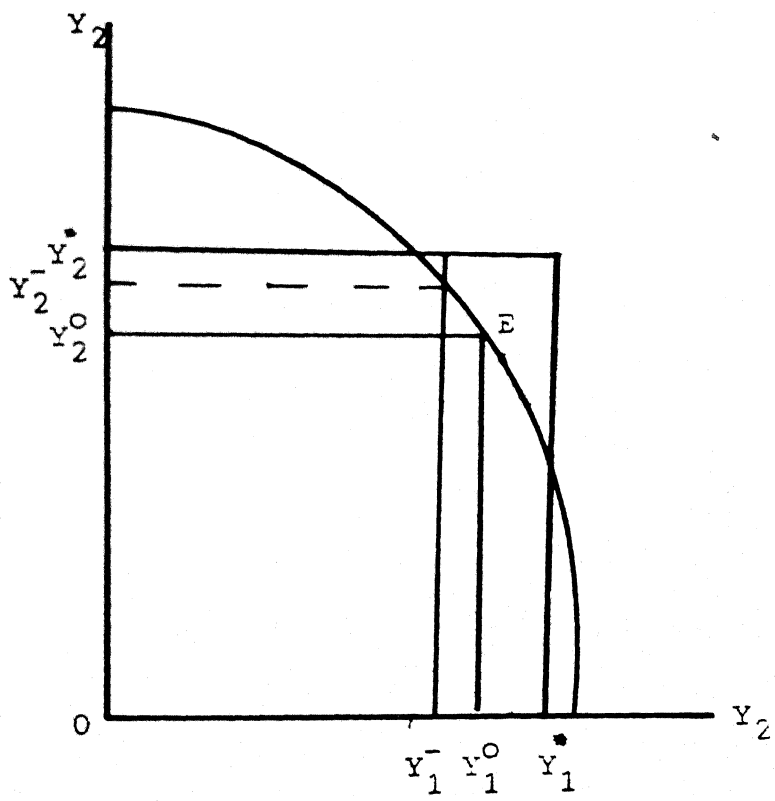


Figure A7

higher than Y_1^O and Y_2^O since the integrability conditions specify that relative shifts in the demand curves D_1 and D_2 have to be equal. The Figures A8 and A9 depict this. If the shift in demand curves is so as to make the value of Y_2 equal to Y_2^O , then also, the point of intersection of $MC_1(Y_2^*)$ and $MC_1(Y_2^-)$ would have to be to the left of Y_1^O . Similarly for Y_2 market and therefore a cost efficient solution would emerge only if the demand curves for both the products are identically the same. Such a possibility seems to be an unlikely occurrence with Y_1 and Y_2 being differentiated in the consumers basket of goods.

If the shift in demand curves is so as to make the value of Y_2 less than Y_2^O , then the value of Y_1 for which $MC_1(Y_2^*)$ and $MC_1(Y_2^-)$ would be equal can be at most at the value of Y_1^O . In such a case however, the value of Y_2 for which $MC_2(Y_1^*) = MC_2(Y_1^O)$ would be to the left of Y_2^O . The ensuing values of (Y_1^O, Y_2^-) are not efficient since for Y_1^O , the efficient configuration is (Y_1^O, Y_2^O) . Hence the result holds. Figures A10 and A11 represent the above arguments. It seems to be apparent that the behaviour of demand as well as costs would have to be far more restrictive if an efficient solution is to be restored.

Case II : Complementary Products

Consider Y_1 and Y_2 to be complementary. Starting again from $C_{12}(.) = 0$ assumption, proposition three shows that the introduction of complementary relations among the products would lead to inefficiency.

(a) Let $C_{12}(.) < 0$ initially. Let \hat{Y}_1 and \hat{Y}_2 be the initial equilibrium values of Y_1 and Y_2 with independent demands. Further

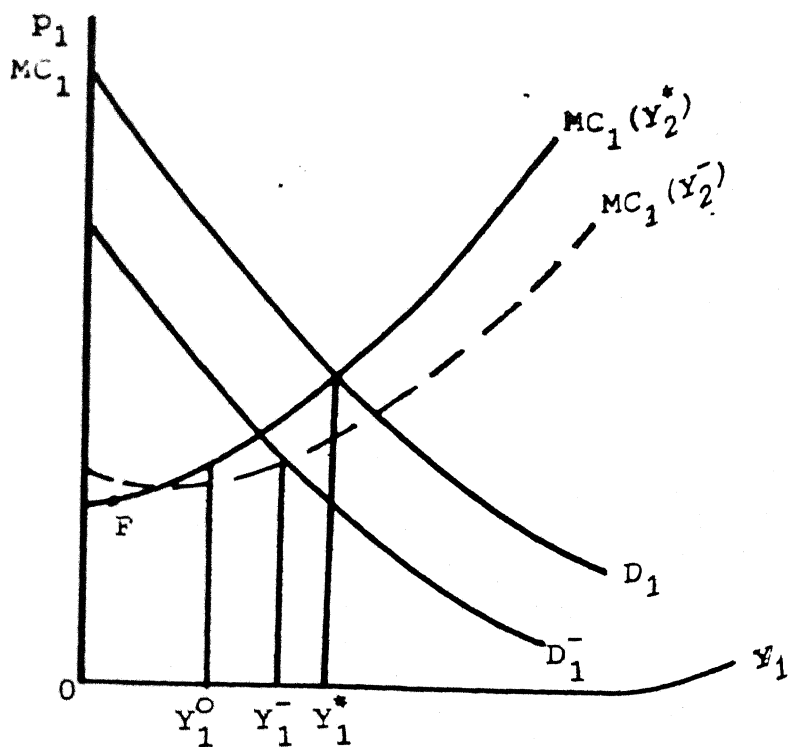


Figure A8

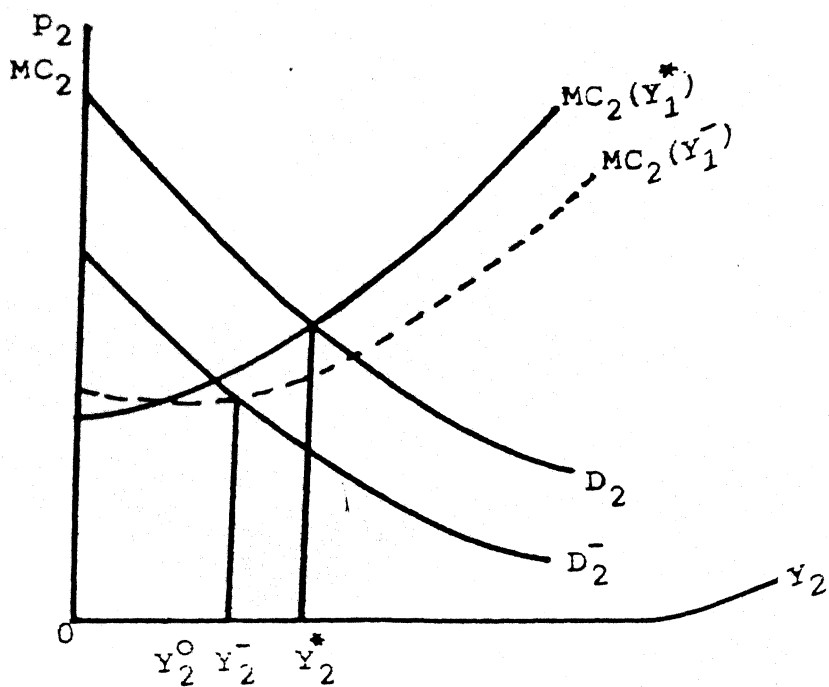


Figure A9

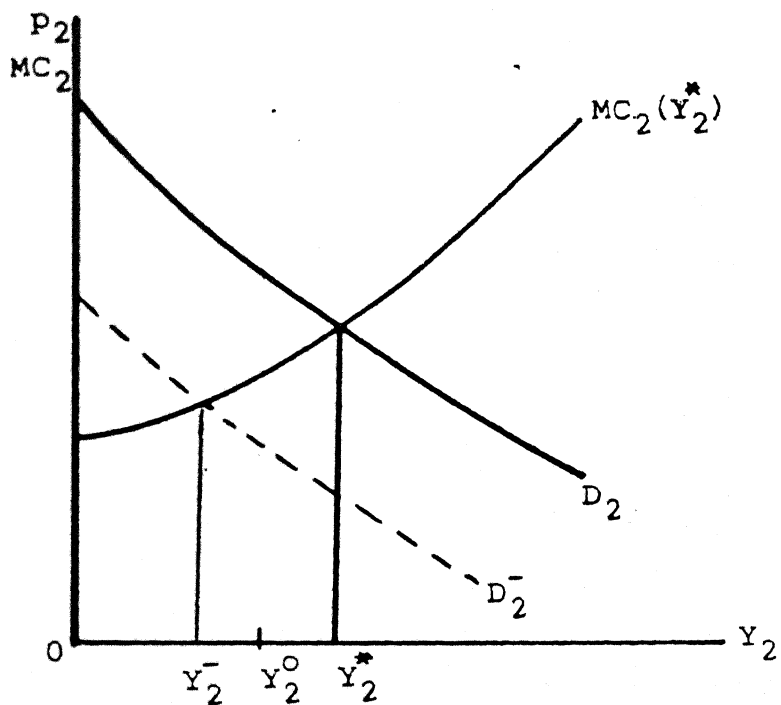


Figure A10

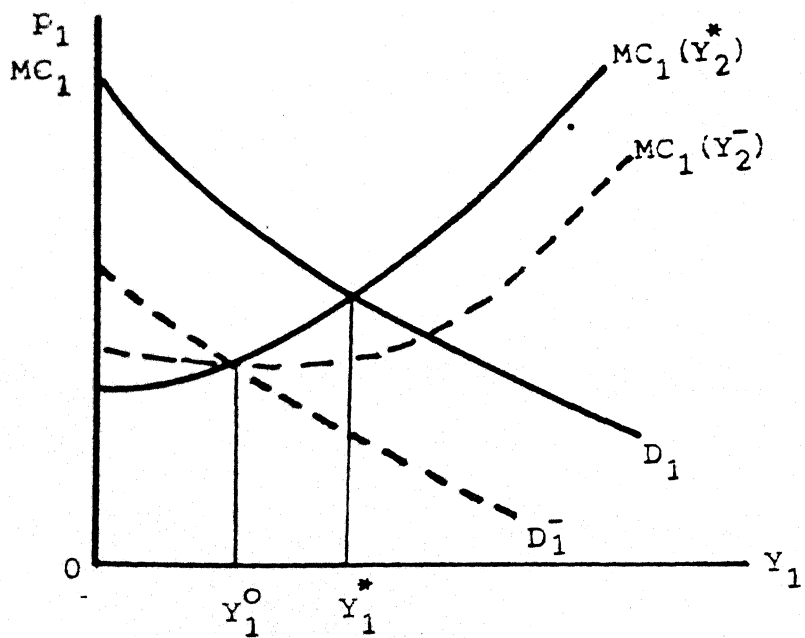


Figure A11

let $\hat{Y}_1 < Y_1^O$ and $\hat{Y}_2 < Y_2^O$. The Figures A12 and A13 show the relative positions of the two equilibria in the two markets. Now, introducing complementary relationships, the D_2 curve (D_1 curve) shifts upwards. Again, the value of $Y_2(Y_1)$ would increase. This increase may be so as to be less than, or equal to or greater than $Y_2^O(Y_1^O)$. The Figures A14 and A15 show the final equilibrium positions for increases in $Y_2(Y_1)$ which are less than $Y_2^O(Y_1^O)$. The results of the proposition three holds only in so far as inefficiency is maintained. If the increase in \hat{Y}_2 is such that it becomes equal to Y_2^O , then the marginal cost of \hat{Y}_1 would be lowered. The point at which $MC_1(\hat{Y}_1) = MC_1(Y_2^O)$ will lie to the right of Y_1^O . This, together with an increase in \hat{Y}_1 when D_1 becomes D_1^+ gives the ensuing result, shown in Figures A16 and A17. An efficient solution can be restored if the quantities of Y_1 and Y_2 deviate from Y_1^O and Y_2^O by the same amounts, and that the shifts in the demand curves are also by these amounts. This cannot be assumed a priori.

(b) Let $C_{12}(\cdot) > 0$ initially. Then outward shifts, of the demand curves, introduced by complementary effects would only serve to move the equilibrium values further away from the boundary values. Hence inefficiency is compounded. The following Figures A18 and A19 show this.

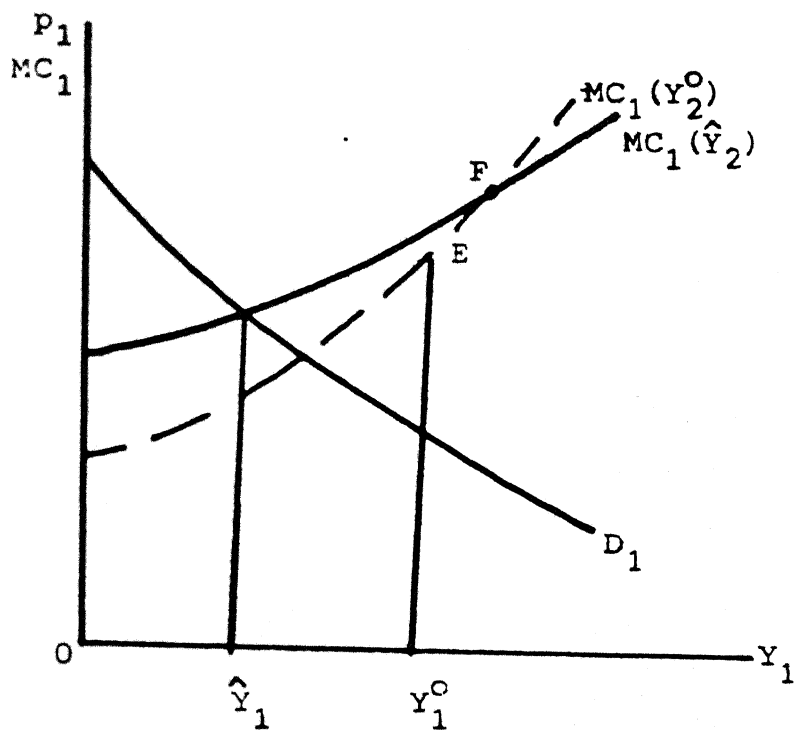


Figure A12

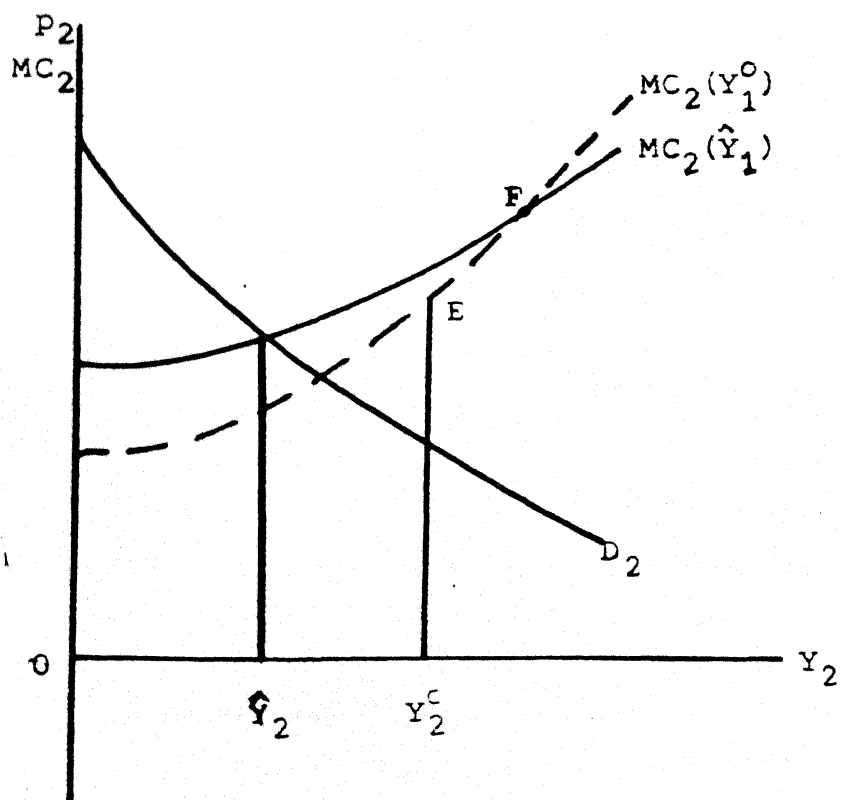


Figure A13

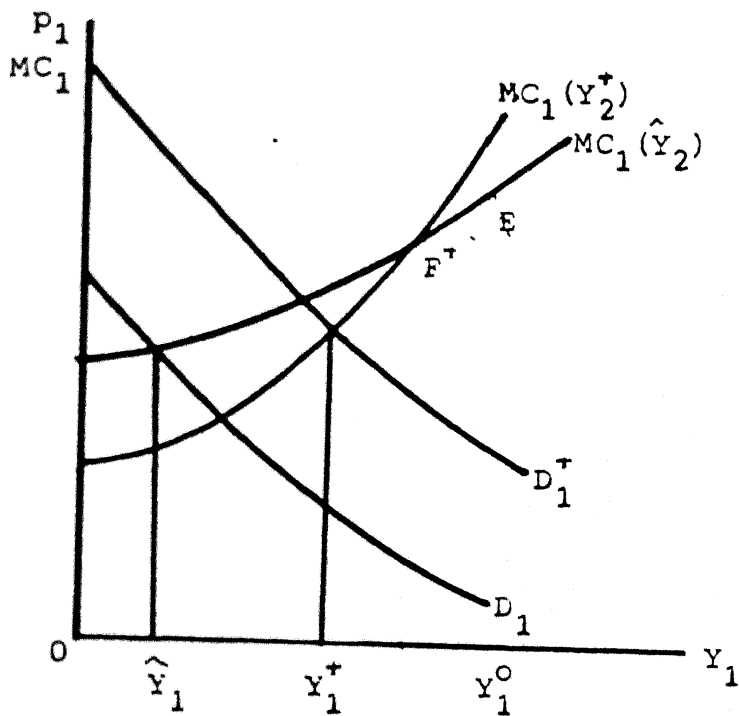


Figure A14

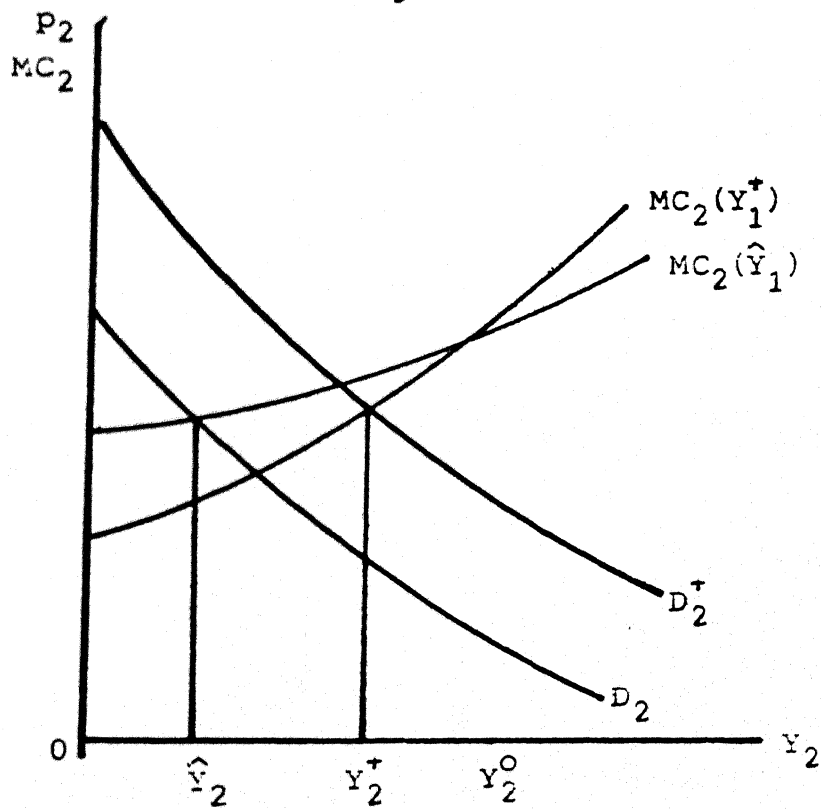


Figure A15

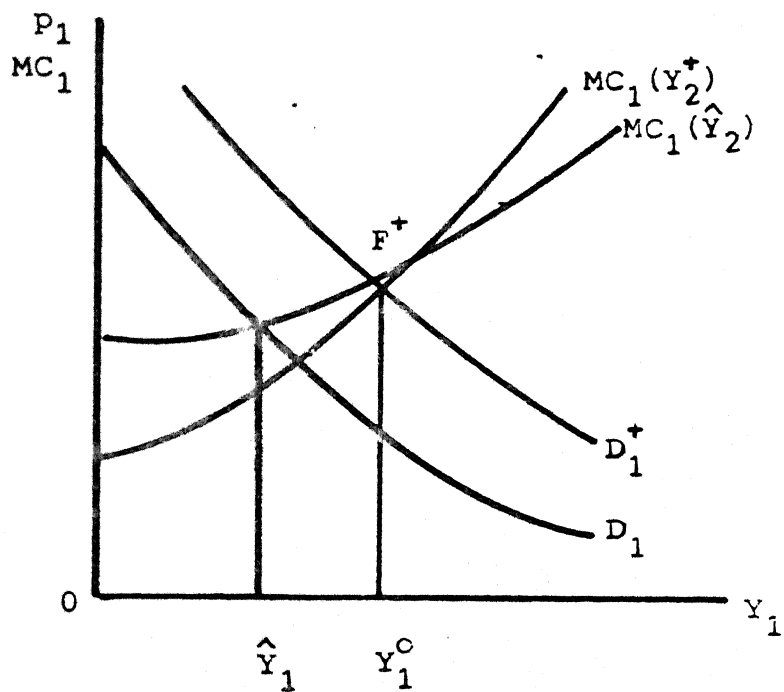


Figure A16

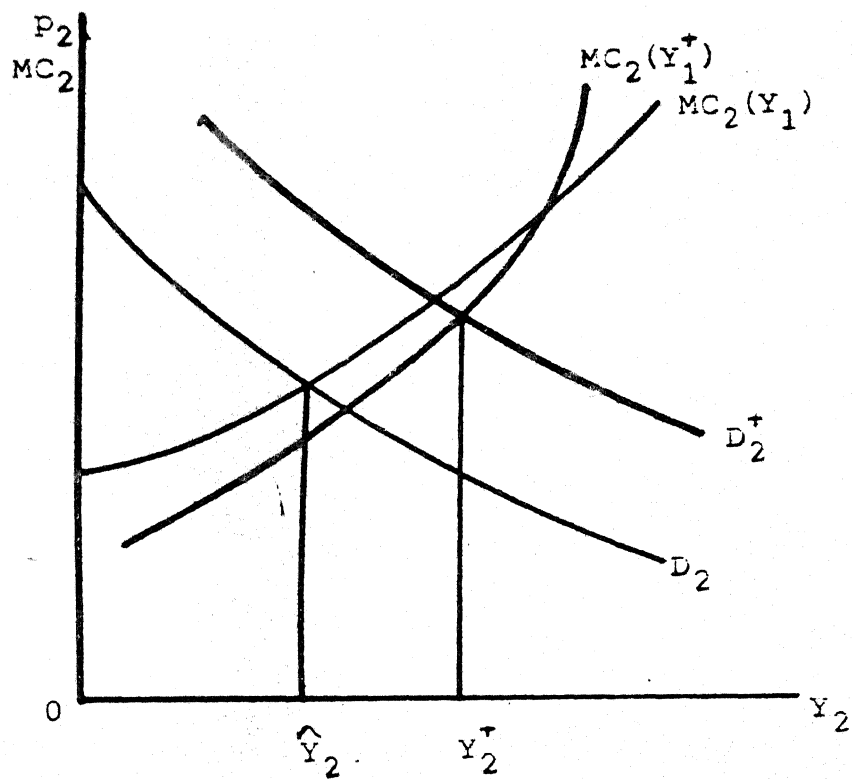


Figure A17

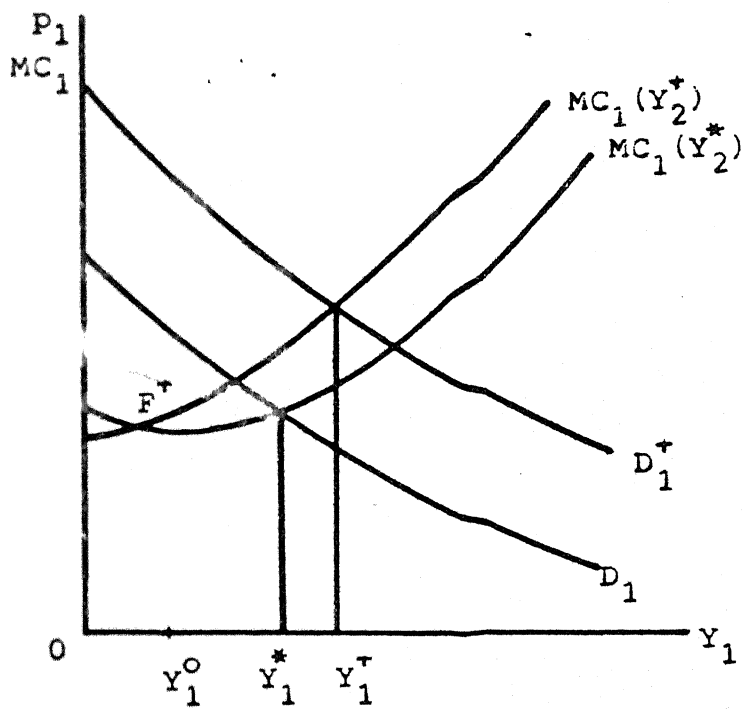


Figure A18

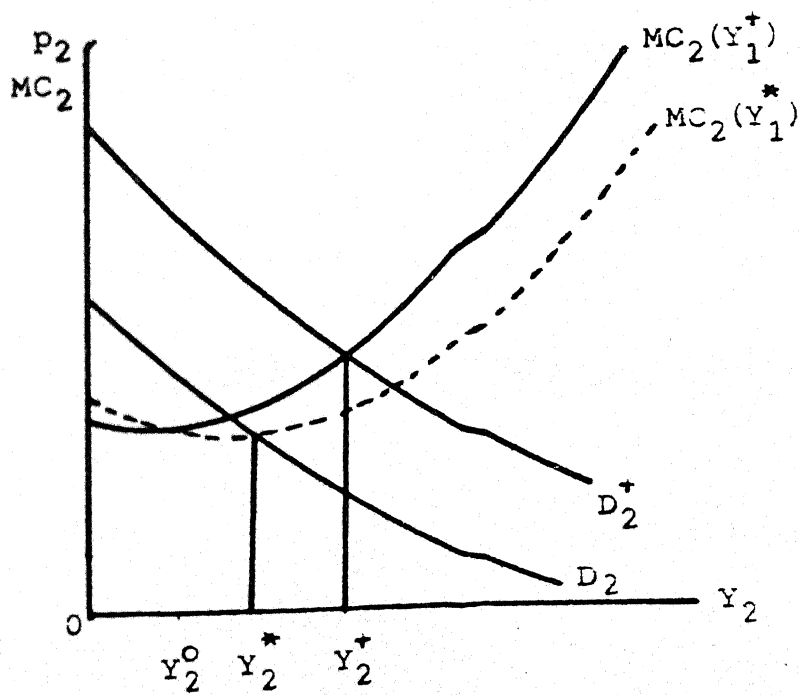


Figure A19